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The influence of previous understanding  
and relative confidence on adult maths  
learning: Building adult understanding on  
a brownfield site.

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# **Abstract**

Expansion of Higher Education has resulted in increasing provision of Access and Foundation programmes, often aimed at mature learners. Adults returning to learn mathematics bring with them a wealth of prior understanding and expectations.

The two common teaching approaches, remedial 'fill in the gaps' or mythical 'start again', are popular with students but argued to be unrealistic because the teaching of adults is better likened to building on a brownfield site. The purpose of this research was to consider what understandings adults brought with them and explore how these understandings interacted with new learning.

203 foundation students were given questions, on proportional reasoning, percentage calculation and over-generalisations. Responses, and response hierarchies, were compared with those from children in the 1970's CSMS survey (Hart, 1981a). Individual behaviours were then explored through interview using a framework developed from ideas of Schoenfeld (1992) and Leron and Hazzan (1997).

It emerged that multiple interactions and choices of behaviour were taking place, indicating final answers, right or wrong, represented only one possibility from a selection of outcomes. Method selection might be influenced by number and beliefs in non-conservation of operation (Greer, 1994) causing potential difficulties for building new learning by method extrapolation. The habit of self-checking and testing for reasonableness might cause difficulties when reasonableness could not be recognised or was counter-intuitive. Other themes identified included: the false recall of certain number calculations with potential for interference with diagnostic practices and the belief in the 'one right method' based on perceived outcome

requirements or confidence from previous success, causing reluctance to consider more efficient or appropriate methods.

This research highlights the benefits of making processes and choices explicit to teachers and students facilitating the integration of previous understandings with new ways of working without disempowerment and increasing the potential for new learning to be built.



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- Overview
- Adults as self-directed learners
- Adult Mathematics Anxiety, Panic and Shame
- Adult self-image
- Adult views of mathematics
- A different adult perspective?

## **Chapter One – Introduction and Rationale**

The continued expansion of the Higher Education sector has been mirrored by an expansion in the range of qualifications acceptable for university entry and a diversification in the routes and courses available to achieve these. Whilst some mature learners wishing to return to education still select the more traditional A-level or vocational qualification routes, others select Access or Foundation courses aimed specifically at mature or international students. Normally a year long, these courses are designed to prepare students for entry to year 1 of a university degree programme and provide appropriate knowledge and skills for a chosen progression route alongside a raft of study skills. Foundation Year courses should not be confused with Foundation degrees which are a different phenomena and involve a partnership between university and workplace.

I am a mathematics lecturer on a university foundation programme which prepares students for a number of different degree progressions including: Primary Education, Biomedical Sciences, Engineering, Business and Human Sciences (see Appendix 2a). No formal pre-qualifications are required for entry, although routes such as Engineering tend to attract those with more confidence and competency in mathematics than routes in non-mathematical disciplines such as Human Sciences.

Adult students returning to mathematics learning bring with them a diverse range of pre-knowledge and prior learning experiences, which are likely to affect their future learning (Knowles, 1980). These may have been gained through home and workplace activities as well as at school and include attitudes and expectations towards mathematics study as well as mathematics understanding itself, whatever that might be defined to be. From a constructivist approach, people build their

new learning on previous learning. Hence, developing appropriate ways for teaching adult returners needs consideration of what that previous learning was and identification of how this consideration should be incorporated into a teaching approach.

It seemed to me to be helpful to consider learning through the metaphor of building on a Greenfield or Brownfield site. In the former, basic facilities such as power, drainage and roads might be in existence but predominantly the builder designs and builds the house from foundation to roof. Completing a partially built house on such a site at a later date simply requires slotting into the correct stage of the building process. Thus, beyond assumptions of basic skills such as addition and multiplication, the teacher attempts to build a new set of learning by either ignoring anything already there or assuming that what is there is the same as their design and can be built on or linked to. This approach of 'start again and fill in the gaps' appeared to be common in adult education.

The notion of a new start is a popular and comforting one for some adult returners, particularly those with high anxiety levels and little confidence in their previous learning. However, as Evans (2000) states, this is clearly a fantasy. Starting from the beginning and covering work that children would cover in a school lifetime in less than a year, inevitably leads to narrower teaching and a very behaviourist approach. One method is taught, which may or may not link with adult memories, rather than an exploration of alternative methods. Teaching is based on the development of one hierarchy of sub-skills selected by the tutor rather than a consideration of multiple alternative hierarchies (Gagné, 1968, 1969, 1985). If a student truly began again and made no links with anything previously learnt, the resulting workload would be immense.

Sometimes students who have managed in the past might be reluctant to revisit or unpick what they might perceive as fragile learning. For them the notion of 'filling in the gaps' or the remedial approach might be more attractive and this is the approach offered by many adult self-study guides. However, this is essentially built on the same behaviourist model as the 'new start' and relies on a belief in one right method or one hierarchy of learning. Increasingly popular in university pre-arrival courses, where there is a growing awareness of issues around larger student cohorts and perceived deficits in existing knowledge, the purpose is usually to ensure everyone is ready for the next stage.

Whilst such courses acknowledge that some knowledge is already there and can confer a level autonomy to learners to identify their own gaps, they fail to recognise the dangers of incomplete or partial knowledge. Nor do they recognise that some knowledge may be incorrect or have been corrupted or distorted by experiences. Misunderstandings and particularly over-generalisations, can slip through the net unchallenged and provide a restrictive influence on future learning. It is my suggestion that building adult understanding is more akin to building on a Brownfield site.

On a Brownfield site, the builder needs first to survey existing structures to identify what could be built on or incorporated into a new design, how one structure might be reliant on another, but particularly what is unsafe or should be demolished. Adults may have a large reservoir of previous knowledge which could provide opportunities for deeper understanding, but not all of this knowledge is 'rich' in the way Knowles (1980) suggests. Some will be incorrect, confused or hold back new construction.

The ultimate purpose of this study was to improve adult teaching practices in response to improved understanding of adult learning and in particular of interactions of old and new knowledge. This led to the research question:

**How does old learning and learning from other situations interact with new learning?**

The underlying assumption for this research was that adults enter a class with a much more diverse set of pre-knowledge than children and this assumption needed to be tested prior to consideration of interactions of old and new knowledge. Mathematics is practiced in many different ways and in many different contexts (FitzSimons, 2005, 2007; Lave, 1988; Maier, 1991; Roth, 1999) There are a wide range of alternative activities with the potential to be labelled mathematics and, therefore, the question needed to reflect the recognition that only a small part of that labelled 'mathematics' could be considered in one study. This led to the identification of a preliminary research question:

**What understanding do adult students have in certain mathematical areas and how does this compare with the understanding of some children?**

A review of literature identified that one of the most influential studies on children's understanding was the Concepts in Secondary Mathematics and Science (CSMS) study that took place in the late 1970s (Hart, 1981a). Much of the work from that study still underpins our awareness of mathematical strategies and methods. Further, it provided detailed analysis of children's responses to specific questions allowing opportunities for comparison. It was decided to use some of these questions for adults.

My discussions with past students had identified that some people, when faced with a large discrepancy between their own pre-knowledge and new learning,



chose to reject their earlier understanding in favour of the new if they had little confidence in their old ideas. In contrast, Skemp (1979) and Swan (2006) had suggested that firmly held beliefs might be more difficult to overcome. For this reason, I decided to incorporate consideration of levels of trust into the research design and to investigate responses from students both before and after they had completed the mathematics aspects of their course. Indications of trust proved helpful in analysing behaviours but when answers were examined in detail and discussion with students took place, it became clear that far more complex interactions were taking place when mathematical questions were being answered. There was evidence of the use of strategies, problem solving techniques, expertise and coping mechanisms at work. At this stage an additional research question was added between the other two:

**To what extent does the mathematical behaviour of adults mirror their understanding?**

In summary, the three research questions were:

1. What understanding do adult students have in certain mathematical areas and how does this compare with the understanding of some children?
2. To what extent does the mathematical behaviour of adults mirror their understanding?
3. How does old learning and learning from other situations interact with new learning?

The first stage of this research was to consider the theoretical background and hence the next section contains the literature review.

## **Chapter Two – The Literature Review**

### **2.1 Three Learning Approaches**

#### ***Introduction***

Until relatively recently, many educators considered teaching and learning together, with the focus on the teaching side of the partnership. In part, this may have been because learning was viewed to be a straightforward process not requiring study. It may also have been because teaching methods or folk strategies were passed on from one generation to the next with apparent success, at least for some students. The notion of knowledge as simply memory deposited in the brain (Sacks, 1999) together with early ideas of memory retention through frequency of association reinforced the perceived value of these teaching techniques. However, the generation of new theories of development, of the mind, of memory and cognition indicated that learning is a much more complex process than simply writing on a blank slate and educators such as Bruner (1966) made the call for the recognition that an understanding of learning was crucial for effective teaching.

The ultimate purpose of this study was to improve adult teaching practices in response to improved understanding of adult learning and interaction with pre-existing knowledge. Therefore, it was appropriate to begin by developing an initial theoretical framework through consideration of learning processes. The field of learning theory was fed by many alternate sub-theories which could be grouped in different ways depending on the aspects of learning under consideration. For this review, the initial groupings selected were based on those proposed by Merriam *et al.* (2007) in their review of adult learning. The four groups of approaches are:

behaviourist, cognitive, humanistic/affective and constructivist. Grouping learning approaches under different headings provided a useful structure to introduce key ideas, but there were few contexts where these ideas were truly mutually exclusive. It was, therefore, not surprising that many key players including Piaget, Vygotsky, Gagné and Bruner featured in more than one section.

## ***Behaviourism***

Behaviourist models define learning as observable changes in behaviour and derive their theories from the notion of stimulus and response identified historically by Pavlov and Thorndike. Through studying animals in repeated experiments, Thorndike (1927) formulated the Law of Effect which showed that responses which were followed by positive consequences were more likely to be repeated than those which were not, and the Law of Exercise which emphasised the role of repetition in improving learning. Skinner (1971) applied these ideas more closely to education and focused particularly on the manipulation of the environment to develop required outcomes.

The behaviourist approach on valuing and measuring observable change is argued by Swan (2006) to still be the dominant model in many school classrooms. Merriam *et al.* (2007) suggested that it is also dominant in adult education, particularly in training courses aimed at providing skills rather than understanding. The emphasis on measurable and observable learning outcomes fits well with current concerns on accountability (Merriam *et al.*, 2007) and is clearly evident in module outlines in Further and Higher Education programmes, including those for the Foundation Programme within this study. It is standard practice for proformas to list learning outcomes in terms of knowledge, skills and key skills and map these outcomes onto assessment activities. Further, student evaluation

questionnaires typically contain questions relating to the achievement of module outcomes. Additionally, personal observation suggests that this emphasis on measurable change fits well with the expectations of students themselves who are sometimes particularly keen to measure their own progress.

Gagné (1968, 1969, 1985) studied learning from a behaviourist approach. He looked at the capabilities of learners before and after learning situations and attempted to identify the environment or conditions that were needed for learning to occur. In planning for learning he asked, 'From where does the student begin; and where is he going?' (Gagné, 1969, p.24). This is a question which remains fundamental for the planning of Foundation teaching today. However, whilst my study focussed on where adult students really begin and what they do know before they return to learn, Gagné's (1968, 1969, 1985) studies focus on what students should know, the pre-requisites for learning. Gagné suggested that in order to reach some specific learning goal, a number of subordinate skills required mastering in a particular order. He tested his hypothesis by considering the capabilities of students who had acquired certain sub-skills with those who had not. He proposed that this order, or hierarchy, bore 'some relation to a plan for effective instruction' (Gagné 1968, p.1).

The notion that it might be possible to consider the capabilities of Foundation students before and after a learning activity, or indeed after the whole course, is an attractive one, fitting well with any behaviourist discourses in the institution. However, unfortunately it is not realistic. The most common set of sub-skills might be readily identified, but Gagné (1968, 1969, 1985), noted that sometimes alternative sets of sub-skills also exist. Assessment would need to ensure that all possible sub-skills were investigated and it would be difficult to ensure that such a

list was exhaustive. Foundation learning is not a controlled experiment, many changes other than simply learning take place including changes in the affective domain. Even if an identical paper was administered before and after learning, the test itself would not be identical because it would not be viewed in the same way by students. Thus a student may have the same lower order skill available for both tests but select alternative problem solving strategies based on new perceptions of desirable outcome (see Section 2.5).

The Gagné influence on learning can be seen in the design of many self-instruction activities, including distance learning programmes and structured worksheets which fragment large tasks into a series of smaller steps providing opportunities for individual practice and feedback. This practice-assessment loop, whether performed informally by students checking answers or more formally by mini tests is essential for progress and is thus designated formative assessment. Whilst superficially 'student centred', since tasks appear to adjust to the particular level of each student, Swan (2006) argued that tasks follow a hierarchy based on the teacher's logic which may not be the most effective strategy for each student. Indeed, Gagné (1968, p.3) himself clearly noted a particular course of instruction would not be the best for every individual student but was only the one that provided the:

most probable expectation of greatest possible transfer for an entire sample of learners concerning whom we know nothing more than what specifically relevant skills they start with.

An example of such an approach on the Foundation Programme is used when solving linear equations. Practice of solving equations of the form  $x + a = b$  is followed by those of the form  $x - a = b$ ,  $a x = b$  and  $x/a = b$ . Competence is then

tested through the solution of a mixed set before proceeding to 'two stage' equations such as  $ax + b = c$  where two operations are required for solution.

## ***Cognitive Approaches***

In contrast to behaviourist approaches, cognitive approaches focus on the internal workings of the mind which cannot be directly observed, although it should be noted that recent advances with Functional MRI and PET scans are beginning to provide some visual clues about the working of the mind. Cognitive approaches assume that the mind actively processes information to give meaning, making use of prior knowledge in this operation. Further, the mind is able to reason and then problem solve (Gluck *et al.*, 2008). Whilst the purpose of learning for the behaviourist is to produce a change in behaviour, for the cognitivist, the purpose is to 'develop capacity and skills to learn better' (Merriam *et al.*, 2007, p.295).

Merged under this umbrella theme are a vast number of overlapping areas of interest which can be grouped in different ways and include theories of instruction as well as learning. The brief overview that follows considers some of the areas found to be most relevant to this study.

### **Memory Storage**

Building on work by Ebbinghaus (1964) investigating the retention of three letter nonsense words, Miller identified that the mind appears to have a limited capacity for remembering strings of numbers. This limit or digit span is between 5 and 9 for most young adults and slightly less for younger children. Later researchers identified similar limits for memorising other forms of information including words and ideas. Therefore, if numbers can be grouped together, arrangements learnt as patterns or information 'chunked' in some way (Glaser, 1999), it is possible to

remember more. This 'short term memory' is transient. It can be maintained by continual repetition but is rapidly lost or displaced by new inputs unless information is retained by storing in some deeper way.

The notion of short term memory has mainly been subsumed into Baddeley's (1986) model of 'working memory' which, as the names suggests, involves not only storing but working or doing. This model includes an executive control which both maintains and manipulates information. Working memory is involved in all forms of reasoning and not only selects and retrieves information from deeper storage but allocates cognitive resources to different tasks. The capacity of working memory is believed to be limited and when this limit is reached there sometimes needs to be a trading process between storage and processing (Just and Carpenter, 1992). Processing may slow down, partially completed results may be forgotten and old information displaced. Thus, someone working aurally through a complex mathematical task may well find they have forgotten the number they started with. For example, Hart (1981b, p.89) described how children in the CSMS survey appeared to carry:

a considerable amount in their heads and were unable to recall all the various segments and thus gave an incorrect answer.

The idea of limited capacity can also be used to identify how processing might be improved. Strategies, such as the chunking process which has just been described, aid 'cognitive economy' (Cowan, 2003) i.e. use up fewer resources and thus help leave cognitive resources for other activities. Storing intermediate answers on paper or within a calculator also frees up resources. Studies on expertise discussed later (Glaser, 1999; Greeno *et al.*, 1999; Schoenfeld, 1992) show that many 'experts' utilise such processes.

For deeper storage, information needs to be actively processed in some way. This may involve specifically linking information to some form of taught memory structure, imagined pegs or rooms as in mnemonic strategies. Alternatively, this may involve expanding the information with a visual or additional thought (see for example Gluck *et al.*, 2008). On the Foundation Programme for example, the link between the 'mode' and the word 'modern' is highlighted to remember which average is which. However, most information is processed by linking it to existing stored information.

Models of the possible storage structure of the brain began with the work of Aristotle who proposed that memory is formed by associations or links between information pairs. Associations were made when two events were close in space and time (contiguity) and strengthened the more often they were met (frequency) or the more similar they were (similarity). A number of alternative models have developed from these original ideas. These new models are sometimes referred to as associationist or connectionist models because they propose various forms of mapping multiple connections linked to others through networks, with complex ideas formed from more elementary ones (Gluck *et al.*, 2008).

Alternative models of the mind have developed from analogies with a computer. However, the so-called, symbol-processing or s-p view has some limitations because no matter how complex the model becomes, it can never mirror every aspect of reality (Bredo, 1999). Further, symbol processing methods are less able to cope in unfamiliar environments and on non-standard tasks. However, the use of the computer as a metaphor for the working of the mind has led to some valuable ideas and helped in the development of theories for complex behaviour. The notion of dual processing (Stanovitch and West, 2000) which is discussed in



Section 2.5 seems particularly helpful when considering some adult behaviour during problem solving.

Piaget (Piaget and Inhelder, 1958) developed further theories of knowledge structures using schemata. These ideas formed the basis of some early constructivist theories and are discussed in more detail in Section 2.2.

### Skills memories

The memories considered so far have mainly been of facts or events, although the storage of ideas or concepts has also been mentioned. Episodic (events) and semantic (facts) memory are sometimes referred to as declarative or explicit memory. In contrast, memory for skills (abilities which improve with practice) are sometimes referred to as non-declarative because they are not always 'acquired and retrieved with conscious awareness' (Gluck *et al.*, 2008, p.126).

Skills may be perceptual-motor skills like eating with a spoon, cognitive skills which involve reason or a mix of both. Studies have shown that the rate of improvement of a skill is greatest at the beginning and gradually reduces over time. However, with enough practice, some skills from walking to simple multiplication to more complex mathematical skills may reach the point where they are performed automatically and with little conscious thought (see for example Greeno *et al.*, 1999). These skills might include rapid recall of number facts such as  $8+2=10$  or  $9\times3=27$ . More complex examples might be the rapid use of an algorithm to add fractions or to solve a linear equation.

Some mathematics practitioners contest whether or not people should be able to unpick automatic processes. For example, Glaser (1999) argues that attempts should not be made to separate them back into component skills. Others,

particularly those teaching people 'starting again', argue against developing skills without understanding (Buxton, 1981). The problem here is deciding whether this means understanding at the time of development, recall of that understanding whenever processes are used or recall that some previous understanding existed. An even larger debate exists, of course, over the use of the ultimate automotive processor, the calculator! It is not uncommon in my classrooms for an adult to describe the use of a calculator as 'cheating' and insist that underlying calculations must be performed or understood in the head before going any further, even though the presence of calculators in the workplace has meant that the need for manual and mental calculations has changed (Wedeg, 2000).

### Memory Loss and Retention

It is apparent from a variety of research sources that memory and associated learning abilities vary with age in healthy adults, although Gluck *et al.* (2008) in particular emphasise the importance of recognising that such work deals with averages not individuals. Working memory declines slightly and new learning appears to take longer in 'older adults' but there is some variation between sources in the definition of the term 'older'. Most often this refers to those over 60. Sometimes it refers to all those not classed as 'young adults', i.e. 17 – 35 years of age (Gluck et al , 2008). Ages within the Foundation cohort vary, but the vast majority are below the age of 35 and hence working memory decline is unlikely to be influential in this study. Indeed, the reverse is true in that the greater working memory of adults over children is more likely to have an influence.

At this point it is important to emphasise the distinction between the effect of age on memory and the effect of time since formal learning ceased. The latter has the potential to be highly influential in this study. Work on retention of unconnected

data (three letter nonsense words) by Ebbinghaus (1964) identified a 'forgetting curve' with the highest level of forgetting being in the first 20 minutes, a slightly lower level of forgetting in the next few hours followed by a gradual flattening out over the next few days. At the other end of the timescale, various studies in non-mathematical areas have suggested that there is little difference in retention between learning from 10 and 15 years ago. 'Information that survives the critical first few days might last in memory indefinitely' (Gluck *et al.*, 2008, p.91).

Since the majority of the Foundation cohort last studied school mathematics at least ten years ago with very few leaving school in the last five years, the school mathematics memories retained could be argued to be fairly fixed. However, memory loss of mathematics met more recently might still be in a dynamic phase.

Semantic memories of facts have been found to be strengthened by repetition and weakened by interference. The observation by Buxton (1981) that adults seemed to be able to recite rules that they did not understand but had learnt by rote supported this. Brase (2002) argues that the mind is pre-programmed by evolution to certain kinds of survival tasks, such as counting, more readily than to others. In adult life, weak memories may be subsumed by these re-emerging naive strategies or by other later ideas. This is termed retroactive interference.

Retention of deeper learning is more difficult to study quantitatively and as Karsenty (2002) notes little research has been carried out into what mathematical learning has been retained after leaving school. From her work, she suggested that adult memories were not evenly distributed over topics but dependant on how often something had been revisited and reused and the length of the acquisition period. Brase (2002) suggested that since some skills are easier to acquire than others and some things remembered more than others, memory is linked to the

amount of effort used to acquire something. Hence, work that was harder to understand is more likely to be retained.

Both Karsenty's (2002) and Brase's (2002) observations fit with the idea that retention is linked to numbers of connections and levels of processing (Gluck *et al.*, 2008) but this is not as conclusive as the Ebbinghaus (1964) experiments. Neither Karsenty nor Brase had access to the initial learning process of the people they studied and, therefore, have no measure of learning input. Adults may certainly recall a mathematical skill and recall that it was rehearsed frequently or difficult to acquire. However, by definition, they cannot recall things they have forgotten and hence cannot know how often these forgotten items were also rehearsed or how difficult they found them at the time. Further, numerous counter-examples have been observed when Foundation adults seem to have a strong memory of learning a topic and the difficulties involved with doing so, but can no longer remember the topic itself.

Karsenty (2002) and Duffin and Simpson (2000) noted that sometimes, when people remember only a part of a process, they appear to fill in the gaps by 'reconstruction'. In Karsenty's (2002) study, some adults started from a few basic memories of linear functions and then generated a whole new set of ideas which whilst incorrect had internal consistency. This resonates with Lannin *et al.*'s (2007) 'repair theory' (Section 2.2) in which people start from correct facts and go on to generate incorrect conclusions. Similarly, Duffin and Simpson (2000) described an adult who remembered only a part of a formal algorithm for subtraction and appeared to fill the gap in her memory by putting in a new incorrect rule of her own. They suggested that her reconstruction failed because she had not fully understood the original procedure and contrasted her outcome

with that of a colleague reconstructing in an alternative situation, perhaps linking with Buxton's (1981) argument discussed earlier. This notion of reconstruction of memory is not unique to mathematics (see for example Draaisma, 2004) but does add a new dimension to my study. Adults on the Foundation Programme will not simply be arriving with mathematics they can remember and mathematics they have forgotten, but also with their own reconstructions which might be unique, unexpected and unpredictable and, therefore, more difficult to counteract.

Clearly some mathematics is retained rather than reconstructed. Measurement of retention through retrieval will always have limitations. Not all that has been retained will necessarily be retrieved on a particular occasion. There appears to be a considerable interest amongst psychologists in retrieval of number facts by children. By measuring latency, the time taken to answer questions, or by observation or self-report, various researchers discuss which number facts are more easily recalled than others and suggest reasons why. These reasons include the size of the numbers, how links have been taught, how often they have been practiced and association with other bonds (Cowan, 2003).

The use of latency measurements for adults can provide some useful insights into problem solving choices and is revisited in Section 2.5. However, it is limited as a tool for measuring retention since it cannot discriminate between non-use of direct recall because information is missing and non-use of direct recall because an alternative method is preferred.

A further problem occurs with the definition of 'forgotten' since sometimes hidden memories can be retrieved with the help of cues. Buxton (1981), for example, refers to adults using new learning to act as a ladder to retrieve forgotten learning. The principle of transfer-appropriate processing suggests that retrieval may also

be slightly better if the encoding and retrieval cues and/or context are similar. While Gluck *et al.* (2008) mention studies with divers in and out of water and with people learning to classical music, it follows that mathematics learnt in the classroom might well be better recalled in a classroom situation. Mathematical methods, dormant and 'forgotten' might suddenly be recalled if triggered by an appropriate question style. A problem like 'if it takes 2 people 6 hours to dig a ditch, how long would it take 4 people?' brings a surprisingly rapid response from some adult returners who have no memory of formal inverse proportion rules or experience of digging a ditch.

### ***The Humanistic Approach***

The humanistic approach which developed from the ideas of Maslow (1970) and Rogers (1983) is based on a belief in the potential of all human beings for growth. From this perspective, motivation for learning comes from within, with a desire to know and become self-actualised. Humanistic writers tend to focus on notions of student-centred learning and empowerment, both of which are frequently linked to writing on adult learners (see for example Benn, 1997). Notions of individual empowerment, student ownership of learning and of self-actualisation have become a cornerstone of many adult learning institutions. On the Foundation Programme new tutors are inducted into a community that holds these values. Sometimes referred to as the 'Foundation ethos', this influences many of the decisions made with regard to teaching and planning and will also be influential in my decisions regarding methodology and ethics.

Knowles (1980), argued that the teaching of adults is very different from the teaching of children and put forward his ideas of androgogy, based initially on four crucial precepts. He stated that as adults matured:

- 1) their self-concept moves from one of being a dependent personality toward being a self-directing human being
- 2) they accumulate a growing reservoir of experience that becomes an increasingly rich resource for learning
- 3) their readiness to learn becomes orientated increasingly to the developmental tasks of their social roles; and
- 4) their time perspective changes from one of postponed application of knowledge to immediacy of application, and accordingly, their orientation toward learning shifts from one of subject-centeredness to one of performance-centeredness.

(Knowles, 1980, p.45)

Knowles' (1980) clear distinction between androgogy as referring to adults and pedagogy as referring to children has sometimes led to an either/or mentality amongst educators. Adults are self-directing but children are dependent. Adults have a reservoir of experience and children do not. Not surprisingly this view is contested by a number of researchers. Merriam *et al.* (2007) for example suggest that in certain contexts children have more knowledge than adults and with new areas of study adults sometimes prefer to be teacher-directed. They argue that:

Pedagogy – androgogy represents a continuum ranging from teacher-directed to student-directed learning and that both approaches are appropriate with children and adults, depending on the situation.

(Merriam *et al.*, 2007, p.87)

However, this argument still suggests a very narrow view of androgogy. For this research, I would argue that Knowles' (1980) set of assumptions should be considered in addition to those already under consideration under the umbrella of

'pedagogy'. In essence, the principles of androgogy can and should be applied to all learners, but in general, are likely to have a greater influence in adult learning.

My observations of Foundation adults, suggest that Knowles' (1980) description of adults as self-directed is very appropriate if adults are considered at a meta-level, rather than in individual learning situations. In a teaching session adults frequently expect to be teacher-led, but take control of their own learning and engage with the process because they want to be there (Duffin and Simpson, 2000). They motivate themselves to study outside the classroom and beyond the topic and are proactive in seeking help. Further, and perhaps increasingly in the current climate, adults have a clear expectation of what the teacher-led process should be and this is often strongly related to their views of teaching from their past, even if previous experiences have not been positive (Duffin and Simpson, 2000). In contrast to Knowles' (1980) fourth point, adults on the Foundation Programme expect teaching to be subject centred because they perceive that to be the purpose of attending the course. Innovative teachers who use games or provide unusual problems within their sessions are sometimes not valued in module feedback. Ironically, if an eventual outcome of this study is to suggest a change in practice, it may be the adults themselves who prevent this change.

Adults also have an expectation of what mathematics itself should be and which aspects are of most value, again often related to past teaching. The debate about the use or not of calculators is an example of this. Schoenfeld (1992) noted that typical student belief includes the notion that there is one right answer obtained by one right method and that every problem should be answered in less than five minutes. Coben (2000) argued that this notion of one right method is widely held amongst adults. She suggested people believe that each operation has only one



standard algorithm that can be used to solve it and, further, that this is usually the algorithm that has been taught in school.

Knowles' (1980) second observation about previous knowledge is subtly different from the cognitive views considered so far, in that it appears to make a value judgement about the students' previous experience. The adjective 'rich' implies that the experience is always a positive resource but Schoenfeld (1992), for example, noted that a knowledge base can also contain misconceptions, misremembered facts and other things which are untrue and Karsenty (2002) and Duffin and Simpson (2000) have highlighted issues with accuracy of reconstructed knowledge. Further, the belief in this knowledge, based on experience of apparent success in the past, may make these ideas harder to change (Skemp, 1979) (see later discussion in Section 2.2). Associative strength of mathematical facts, confidence in answers and remembered algorithms are all linked to previous experience and may all be instrumental in strategy selection in the future (see earlier discussion on retrieval). In summary, the accumulated 'reservoir of experience' contains not just the mathematics itself but also the so-called affective processes, belief, attitudes and emotion, towards that mathematics. The passage of time and influence of home, work and adult life in general result in a very different starting point for adults from children.

### Emotion

Since the early work by Tobias and Buxton (1981) on mathematics anxiety, considerable research has taken place on adults' responses towards mathematics. Early work was somewhat anecdotal, but descriptions of behaviours which might be symptoms of anxiety such as Bibby's (2002) passing and self-denigration have found real resonance with the experience of many mathematics educators

including myself and the notion that mathematics can engender strong emotions in people, both positive and negative, is now firmly embedded in adult education.

Many writers (Bibby, 2002; Buxton, 1981; Safford, 2000; Walen and Williams, 2002) suggest that there is often a negative correlation between anxiety and achievement, although as Evans (2000) points out this view of anxiety interfering with performance is a dominant view but not the only one. He describes an alternative view, where low levels of anxiety actually improve performance and there is some optimum level of anxiety which maximises success. Certainly, anxiety due to the awareness of the additional importance of a particular task such as an assessment might increase checking and self-monitoring processes and raise confidence thresholds (Siegler, 1988) encouraging the use of back-up processes rather than relying on intuitive methods or direct recall. These ideas are developed further in Section 2.5. Nonetheless, anxiety is generally perceived as having a negative impact on performance.

Buxton (1981) particularly identified severe anxiety, which he labelled as panic. Other studies have since identified a range of emotions such as shame (Bibby, 2002). Many examples in literature focus on small intervention projects for people with self-confessed mathematics anxiety and, hence, I would argue against the over-generalisation that all adults are anxious about mathematics or dislike challenge. Research on adults' mathematics memories by Karsenty (2004), for example, indicated a more diverse range of views. She was able to classify her participants' views of mathematics into five categories, only one of which indicated anxiety. The other four were: exciting and enjoyable, challenging but manageable, unimportant and generating indifference and general lack of interest in any school learning. Evans (2000) argued that some of the statistical analysis of research on

anxiety is potentially flawed and there is a lack of recognition of the influence of sample characteristics. Anxiety levels will be stronger in some groups than others.

It is not clear whether mathematics anxiety really increases with age as suggested by a few researchers or whether some adults are simply more willing to voice their concerns than children. More recently, Klinger (2007) investigated anxiety and attitudes of Foundation and Access students in comparison with undergraduate students (248 valid responses), using detailed factor analysis to consider potential influences of gender, schooling etc. He confirmed that, as he had expected, levels of mathematics anxiety and negative attitudes were higher for women than men, for those enrolled on arts and humanities courses than science courses and for those educationally disadvantaged (i.e. many Foundation/Access students). Further, he argued that early learning experiences were at least as important as gender influences.

Walen and Williams, (2002) identified that the emphasis on speed and efficiency raised anxiety and appeared to devalue thoughtfulness and thoroughness. Higher levels of abstraction and meta-skills can result in greater efficiency which in turn increases speed, but the aim is not the speed but the skill. Numerous stories exist of the negative emotions felt when testing tables in class and the perceived public humiliation linking again with Bibby's (2002) notion of shame. The description by Walen and Williams (2002) of a girl 'freezing' when doing multiplications in a timed task shows how the concern about time interfered with the ability to operate.

Walen and Williams (2002) argued that the negative responses exhibited by two people they observed were not directed towards mathematics or assessment but towards the notion of performing within a set time. Indeed, they showed that their performance improved when they were 'allowed longer' even though they didn't

actually end up using the extra time. The potential impact of set time on performance may need to be considered within my research design.

### Belief and Attitude

Some ideas on belief and attitude have already been identified in the earlier part of this section. However, consideration of some of the adult literature available on affective processes provides some additional insights. Most of this literature focuses on case studies and discussion with people as they work through some mathematical tasks. Examples include Buxton's (1981) work with professional people, Coben's (2000) work on adult life histories and Bibby's (2002) work with primary teachers. Ideas that emerge include the concept of a brick wall or a point at which things did not make sense (Coben, 2000; Karsenty, 2004), the influence of a teacher, parent or 'significant other' (Coben, 2000; Karsenty, 2004; Safford, 2000; Sewell, 1981), the perceived power of mathematics and importance as a gatekeeper (Bibby, 2002; Karsenty, 2004; Safford, 2000; Sewell, 1981). All three might help explain why some attitudes towards mathematics have developed. However, I suggest that it is two further ideas: negative self-image (Duffin and Simpson, 2000; Karsenty, 2004; Klinger, 2007) and low levels of self-efficacy (Bandura, 1994, Safford, 2000) that are of particular interest for Foundation study because they have the potential to be most readily changed.

Bandura (1994) used the term 'self-efficacy' to describe people's judgement or belief in their own competencies. Crucially, he showed that perceived levels of self-efficacy contributed to motivation, performance and, hence, achievement. Thus, adult teaching practices that focus on improving self-efficacy could, ultimately, improve learning. Bandura (1994) suggested that self-efficacy was influenced by experiences of success and failure. Hence, structuring adult

teaching to maximise opportunities for success, mitigating any previous school experiences, would lead to improvement, as described by Safford (2000) within her adult algebra class. Coben (2000) noted that levels of self-efficacy in some adults were such that they believed if they could do it, it could not be mathematics. She distinguished between visible and invisible mathematics, suggesting that students might gain confidence if the common sense mathematics they could do was visibly labelled mathematics, fitting with this notion of emphasising success.

The move of adults towards ownership of learning and empowerment (Knowles, 1980) identified earlier, might be influential in reducing the perceived power of previous criticism. There may be a belief that a 'past self' might not have been able to do something but perhaps, a 'new self' can. The determination to succeed in itself may be influential in decisions to continue with a task rather than give up, which in turn may lead to success and improved self-efficacy. Bandura (1994) also noted the influence of 'social modelling', of people identifying role models and equating their own possibilities of success with the success of their role model, an idea firmly embedded in the use of case studies within Foundation publicity and in decisions to allocate previous successful Foundation students as mentors.

The three groups of learning theories discussed so far, behaviourist, cognitive and humanistic, each contribute towards developing a theoretical framework for this study. Each refers to previous knowledge in some way. For Gagné (1968, 1969, 1985) the correct pre-requisite knowledge was essential for the next piece of learning, although there was some recognition that there may be more than one set of subskills that could provide this. For the cognitivists, new knowledge links to old knowledge. Considering adults from a more humanistic approach, previous knowledge is a 'rich' resource for learning (Knowles, 1980, p.45) or

possibly a mix of affective beliefs (Evans, 2000), correct and incorrect information (Schoenfeld, 1992) and reconstructions (Duffin and Simpson, 2000; Karsenty, 2002) The next section moves on to consider possibly the most influential group of learning theories for my research, constructivist approaches, all of which focus strongly on the building of new knowledge through interaction with the knowledge already there.

## **2.2 Constructivist Approaches**

### ***Introduction***

Constructivist theories consider learning as a process of constructing meaning. There are many different, and sometimes conflicting, groups of constructivist theories with emphasises on different ways of building and different interpretations of the reality of what is built. However, the notions of individual construction and interaction with previous knowledge are the cornerstones for all constructive theories. Whilst the learning theories considered so far in this writing all contribute to development of my research, the principles of construction and importance of previous learning form the overarching principles on which this research is built.

The three groups of theories selected for discussion here have particular influence on my work. They are: Cognitive Constructivism (particularly the work of Piaget), Social Constructivism (particularly the work of Vygotsky) and Radical Constructivism (von Glasersfeld)

### ***Cognitive Constructivism and Schemata***

Piaget and others developed theories on knowledge structures by adopting the pre-existing notion of schemata (Piaget and Inhelder, 1958; Skemp, 1979). A schema is a group of concepts put together in a certain way which can function

passively to integrate existing knowledge, progressing from simple sensory-motor schema, when very young, to increasingly abstract schema later in life. Schemata enable the efficient storage and retention of information. Understanding is built by assimilating new concepts into an existing schema. If the existing schema is not capable of assimilating a new idea, i.e. if the new information does not 'fit', the process of accommodation occurs, where the schema itself undergoes structural change in response to the 'cognitive conflict'. This process of incorporating a new idea and adjusting a schema to accept it is labelled equilibration (Piaget and Inhelder, 1958). The process of cognitive conflict followed by equilibration is, in effect, the process of learning.

Inevitably, the schemata of mathematics students undergo accommodation on numerous occasions, but certain instances of cognitive conflict, particularly those deriving from the extension of domain from whole numbers to fractions are of major importance. The notions that multiplication always gives a bigger answer or multiplying by 10 just adds a nought are amongst a sub-group of conceptions sometimes referred to as over-generalisations. This is revisited in Section 2.6.

Schemata can also function as a 'mental tool for the acquisition of new knowledge' (Skemp, 1979, p.39) by providing an expectation of what is to be found. If the mind is prepared, then more can be stored. Ausubel (1967) suggests that schemata can act as advance organisers to prepare for new work. In a sense then, for adults, previous learning, even if partially incomplete, might help to retain new knowledge. Unfortunately, if the partially incomplete learning is also incorrect, the converse must also be true. Old incorrect learning might hold back the understanding of new work. Further, if ideas have been held for a long time and appear to have held true in previous situations, schemata may be particularly

resistant to accommodation preventing the acquisition of new knowledge. For well developed schemata, and this of course particularly includes adults returning to learn mathematics, Swan (2006, p.75) noted the following:

People whose schemata are threatened will attempt to defend positions, dismiss objections, ignore counter-examples, keep logically incompatible schemata segregated, and so on.

Thus for adults in this study, previous incorrect ideas need to be examined and corrected if new learning is to take place. However, correction is likely to be problematic.

### ***Social Constructivism***

Although some of the most frequent occasions for accommodation are provided by interactions with others (Swan, 2006), Piaget's work considers the construction process to be primarily personal and internal. In contrast, Vygotsky's (1978) view of learning places the emphasis on social interaction as the primary process for making meaning. Individuals actively process information from social interactions with the environment, with teachers and with peers to develop their own representation of knowledge. The body of cultural and historical knowledge is transferred between people and through generations via shared communication, through the use of language both verbal and written.

To help explain the transfer process, Vygotsky (1978) focused on the difference between what a person can do on their own and what they can achieve with the help of more experienced others. He labelled this the Zone of Next (or Proximal) Development, ZPD. Shared activity in this area enables people to interact with the skills of others and appropriate new learning.



In the school classroom, the learner is the child and the more experienced other is usually the teacher. The teacher's role involves identifying the ZPD for each child and designing activities and providing scaffolding (Wood *et al.*, 1976) to enable effective learning. Alternatively, there may be occasions when peers collaborate on a task in such a way that individuals temporarily develop differential expertise and are able to scaffold each other. Merriam *et al.* (2007) suggest that such learning involving personal exchange and negotiation is particularly compatible with adult learning. However, in practice there are some difficulties with this in a mathematics context, in part, because of the uneven spread in expertise and variation in previous experience. Feedback about group working by Safford (2000), for example, showed that some people found it helpful to explain their ideas to others but frustrating when they wished to take time and think things out for themselves. My own observations suggest that sometimes other students are 'false experts' attempting to transfer their own incorrect or limited understanding to others, sharing 'quick fixes' which cause students to reject tutor teaching but cause problems later.

Vygotsky's (1978) work implied that successful interactions rely on assessment of where a student is and where they could be and this is not normally part of the agenda of fellow students. Many students ignore the pre-knowledge of those they are 'helping' or assume that others will have followed similar learning paths to themselves or have the same priorities. However, I suggest that there is a danger that tutors too might make incorrect assumptions or assume that a particular response indicates a certain block of pre-knowledge.

There are certainly occasions when two students form successful learning partnerships, working together on worksheets and topics both inside and outside

the classroom and supporting and scaffolding (Wood *et al.*, 1976) each other sometimes throughout their study careers. Additionally, well designed worksheets can sometimes lead a group to a similar place so that constructive discussion about a particular problem can occur. More often though that constructive discussion takes place in the presence of the tutor, who is continually modifying the agenda in response to the contributions by the students and her/his assessment of current understanding.

Such formative assessment is an essential pre-requisite for success but is not a simple matter. Unlike formative assessment for Gagné instruction methods, the purpose is not to check for competence in specific pre-skills. Instead it is to gauge a student's position on a learning path. Adults will have a much wider range of pre-experiences than a group of children and adult learning paths will have fewer similarities than those of a group of children who have been taught the same topics in the same order. Indeed, the assessment needs to be continual as small advances through guidance or references to alternative contexts might suddenly trigger memories forgotten and a corresponding shift in the ZPD (see earlier discussion on memory retrieval).

Vygotsky's (1978) work centred on learning through activity. Earlier discussion has already noted that some things are easier to remember in the context within which they were originally learnt. However, Vygotsky goes further, suggesting that the learning needs be considered as a function of the context. These ideas of activity theory or situated cognition are particularly relevant to mathematics.

Lave (1988) studied the behaviour of supermarket shoppers and identified that, not only did people use far more qualitative methods for shopping than might have been expected, but that arithmetic activities were carried out in a very different

way and much more successfully by the shoppers in the shop than when presented as a formal mathematics problem on paper. A number of writers (see for example Roth, 1999 and Maier, 1991) looked particularly at the differences between school and 'other' mathematics. Key differences included: school problems tending to be clearly defined but problems outside school being complex and having to be framed as problems before they can be solved, information for school problems being presented ready for use but information outside school having to be selected and extracted, using pencil and paper and/or calculator and standard algorithms for school problems but using oral and mental methods and non-standard approaches for real-life problems. Finally, formal problems seem to require just one best answer but in real life, 'some choices are better than others' (Roth, 1999, p.14). Approximations or probabilities are often sufficient for purpose and are generally a more efficient use of time. Maier (1991, p.63) summed these up by talking of people 'doing parking lot mathematics in parking lots'.

Other studies on the mathematics practised by groups of people in different work also support the notion that arithmetic activities are carried out in different ways in different contexts (see for example FitzSimons, 2005) and thus mathematics practice cannot be considered independently from the context in which it is being practised. Not only is the mathematics itself embedded in the context but the tools to complete the work, strategies for completion and desired outcome differ for different situations. This has led to the creation of the term 'situated mathematics'.

Foundation adults will have used mathematics in school, at work, in everyday life and now on the Foundation course, so the notion that mathematics is situated has major implications for both my teaching and my research. If understanding is to be

measured through solution of mathematics problems, in which context should those problems be set? Is one context of more value than another?

Some help for this dilemma can be found by returning to Vygotsky's original premise that social interaction is the primary process for making meaning. Lave and Wenger (1999) studied such interactions when apprentices were inducted into their trade. They introduced the term, 'Community of Practice' to describe a group of people who share a common interest or profession. Each community of practice has its own set of values, ways of working and desired outcomes: a school calculation of circumference would involve  $\pi$  but a plumber might simply multiply a diameter by 3 for an acceptable approximation.

Children leave school to join many different communities, at home, in varied workplaces and/or in academic study. Each of these communities will have its own set of valued practices producing a major dilemma about what should be taught in the school classroom, and, as a function of this, how it should be taught. The differing priorities of stakeholders, government, academics, employers, educators and the children themselves produce different desirable outcomes. As a consequence, a somewhat hybrid version of mathematics is taught in school and adults can inherit a set of confused priorities about what is important.

For adults joining the Foundation programme, the purpose and outcomes are much more clearly defined and a clear Community of Practice can be identified. The purpose of the Foundations Programme is to prepare students for future study on specific degree programmes i.e. to induct adults into the academic community. The mathematical practices to be valued are those which will be in use on their

progression degree. For the majority of students in this study, mathematics will appear in standardised problems and be assessed through pen and paper testing.

### ***Radical Constructivism***

Radical constructivists (von Glasersfeld, 1995) explicitly state that it is never possible for humans to truly know mathematics in an ontological way, i.e. to know how things really are. Since it is not possible to compare or match individual constructed knowledge with absolute reality, the concept of validity is not an appropriate one for evaluating what has been built. Instead they use the notion of testing for viability, whether something appears to work or fits. Further, they argue that teaching processes such as discovery learning (Bruner, 1966) which are based on assumptions of mathematical truth to be uncovered are flawed (Confrey, 1991). Constructivism is more involved with inventing the truth.

Setting aside the philosophical debate about the existence or not of one absolute truth that is mathematics and passing for now the discussion on exactly how any existing body of mathematical knowledge might fit with this one truth, induction into a university 'Community of Practice' (Lave and Wenger, 1999) requires acceptance of some agreed norms. Whether the tools of mathematics in use have been invented, discovered or negotiated and whether a better set may evolve or previous ideas be proven unsatisfactory in the future, they remain the current version of reality and as such are given value. Much of the progress of humankind relies on understanding, in some way, the existing body of knowledge and then 'standing on the shoulders of giants' to extend further. Indeed this is the accepted model for postgraduate qualifications such as the Ed.d. It is this view of reality that takes precedence for me in my current environment and hence in my interpretation of how and why I teach Foundation mathematics.

Nonetheless, some great leaps in human progress have been the product of alternative ways of thinking 'outside the box', broadening the boundaries or challenging conventions. The decision to focus primarily on standard or narrower activities in my Foundation teaching does not mean that I would not introduce more open-ended and challenging activities in other adult learning arenas.

The most significant influence of the radical constructivist approach for both my teaching and this research has been its emphasis on trying to interpret learning construction through the eyes of the learner by imagining how that learner might be viewing the problem. Using the premise that:

When students genuinely engage in solving mathematical problems, they proceed in personally reasonable and productive ways.

(Confrey, 1991, p.111)

work that had previously been just labelled 'wrong', or perhaps, worse, 'a good try', now becomes something of value to be interpreted. For example, Lannin *et al.* (2007) described how a student can start with a correct procedure and then make their own adaptations (or 'repairs') for a new situation which although incorrect may seem sensible to the student. Lannin *et al.* (2007) referred to this idea as 'repair theory'. For a teacher, a willingness to listen and to try to interpret other ways of thinking leads to a more empathic approach. For the student, the notion that work is of value to the teacher, is labelled as a partial success rather than a failure, leads to empowerment and increased self-efficacy (Bandura, 1994). Successful problem solving, discussed elsewhere, noted that belief it was possible to succeed was influential in the achievement of that success, thus this whole process of giving value contributes to a cycle of positive reinforcement.

Attempting to understand students' ways of thinking allows teachers to guide and

redirect constructively or to link individual mistakes together for more effective remediation (Confrey, 1991). For the researcher, interpreting a learner's actions by imagining their thought processes has the potential to provide powerful new insights. Work by Leron and Hazzan (1997) and by Duffin and Simpson (2000), for example, considered elsewhere in this chapter attempt to use this process.

### ***Approaches to Learning Conflict***

Discussions so far have focused mainly on the building of new learning. Yet it has been recognised that the knowledge base of adults might contain misconceptions, misremembered facts, idiosyncratic reconstructions and things which were untrue (Duffin and Simpson, 2000; Karsenty, 2002; Schoenfeld, 1992). It has been suggested that these ideas might hold back learning and, further, that they might be resistant to change (Skemp, 1979, Swan, 2006). As a tutor, I would suggest these ideas cannot be ignored, but the work of Duffin and Simpson (2000) introduce some new ideas about the responses of students which resonate strongly with my own observations of adult learning.

Duffin and Simpson (2000) considered the issues of integration of old and new learning for adults through the viewpoint of the learner. To do so, they created a new framework which in some ways appeared to be an adaptation of the accommodation/assimilation model. They suggested that new experiences could be categorised in three ways, as natural, conflicting or alien.

In a natural experience, new information fitted in and was assimilated into existing knowledge structures without cognitive conflict. In doing so it reinforced existing ideas and current ways of thinking. For example, if a new teacher suggested that 35% of 40 could be found by the calculation  $0.35 \times 40$  and this corresponded with the method remembered from school, this new information would be natural.

In a conflicting experience, the new information conflicted with that already known. If a student believed that  $35/100 \times 40$  was the only correct calculation, the alternative suggested by the teacher would be seen as a conflict. The resultant cognitive conflict could eventually lead to a merging of ideas in a deeper understanding, perhaps a recognition that  $35/100$  and  $0.35$  were the same.

An 'alien' experience was one where new information 'does not fit and appears not to connect with anything we have learnt before' (Duffin and Simpson, 2000, p.85). The response to an alien experience was to ignore it, avoid it by going back to something familiar, or learn it as something totally new and separate. Using the example just given, the student could ignore the new method, elect to use their original method for the next worksheet or decide that they would use this method for the questions on the current worksheet and for foundation type questions and use their other method for other problems, i.e. not connect the two.

The identification of an 'alien' category provides for me the missing link between accommodation and assimilation and fits with the process suggested by Swan (2006) who noted that people who felt their trusted schemata to be under threat sometimes elected to keep logically incompatible schemata segregated. Similarly Black (1999, p.122) noted that people might be able to, 'play back the 'correct' science explanations in formal tests' but resisted changing their everyday views. This might provide one possible explanation for why someone appears competent with a topic in a lesson and then uses an alternative confused method a few weeks later. If all new information was categorised as natural or alien, it might be perfectly possible to strengthen an existing structure of knowledge with the natural experiences, build an entirely new building of new knowledge with the alien experiences and totally ignore all the old confused knowledge in the middle. Along



the way of course, a potential opportunity for developing a strong interconnected web of knowledge would be lost and the old confused knowledge might still reappear to cause future problems.

One particular set of 'misconceptions', the so-called, 'over-generalisations' which were introduced in discussions on cognitive conflict earlier are worthy of revisiting, in part because the discussion about these highlights some of the differences between learning approaches in dealing with flawed knowledge.

### Over-generalisations

For those teaching mathematics, there is some freedom of choice about the best way to approach a topic. However, given the hierarchical nature of mathematics, which requires tools from one stage to do the next, initial encounters often use simplified examples or are deliberately structured to use only the tools available so far. Hence, early examples might encourage the development of certain ideas, for example: multiplying always gives a bigger answer, subtraction involves taking the smallest number from the largest, multiplying by 10 means adding a nought, and so on. Inevitably, this causes the possibility of over generalisation by some students in the future. Since Swan (2001) argued that students make their own alternative meanings for mathematics all over the world whatever the approach, it is not surprising that discussion of these common errors and interpretation of students' understandings continue to feature in a considerable volume of literature. Alternative approaches from different research disciplines include the description of obstacles or 'temporarily incomplete or faulty conceptions' which, 'produce correct responses within a particular frequently experienced context but not outside it' (Lithner, 2008, p.271) and 'bugs' (Brase, 2002) that need curing.

The use of the words 'misunderstanding' and 'misconception' are sometimes

contested. For example, Swan (2001) argued that these terms imply a clear boundary between right and wrong thinking and encourage the belief that they are undesirable or to be prevented, or 'a medical condition deserving of a particular treatment' (Lannin *et al.*, 2007, p.44). Swan (2001) talked instead of alternative conceptions, well constructed ideas which differ from the culturally accepted ones. Radical constructivists argue that it is wrong to label something an alternative conception or misconception because all ways of thinking are valued and these terms imply one view has higher status than another (Confrey, 1991).

Confrey (1991) argued that a theory cannot be labelled a misconception until an area has been encountered where it fails. Swan (2001) argued that the error is not in the concept but in the failure to recognise the domain over which the concept is valid. Thus the problem of the over-generalisation of 'Multiplication Makes Bigger' could be repackaged to students as, 'When does 'Multiplication Makes Bigger' hold true?'. Further, Swan (2001) considered these ideas a natural part of development to be made explicit and used to develop learning. Many of the problems met in adult life are of a standard type or can be solved with naive strategies. Stating that a rule is incorrect can cause real conflict if a person has been successfully operating with this rule for years. It may cause someone to doubt other mathematics knowledge they have and undermine confidence and self-efficacy (Bandura, 1994). In contrast, using Swan's ideas and highlighting the success, that the idea is correct although only in certain domains, has the potential to both reinforce and extend.

It has emerged from the discussion so far that, whatever the learning approach, 'the reception of new knowledge depends on existing knowledge and understanding' (Black, 1999, p.121). In the last few sections the focus began to

shift towards the role of the teacher within the learning process and particularly the need to clarify this existing knowledge and understanding in order to design effective learning interactions. The next section moves on to discuss the clarification process, assessment.

## **2.3 Assessment**

### ***Introduction***

One label 'assessment' is frequently attached to two different processes generated for two different purposes. 'Summative assessment' is the label given to processes designed to measure attainment against some external criteria or to act in a gatekeeping role. 'Formative assessment' is the label given to assessment to improve learning and as such forms an essential and integral part of learning activity. Both forms of assessment are used on the Foundation Programme and have potential value for this research.

### ***Formative Assessment***

Formative assessment on the Foundation Programme generally occurs in two forms. In the first, the students are explicitly aware of the process and products and may sometimes be responsible for generating and monitoring themselves. In the second, the process and consequences of that process are hidden from students since they take place within the mind of the tutor. The former links to Gagné's (1968, 1969, 1985) ideas of ensuring that required sub-skills are developed before proceeding to a new task that requires them. The latter fits more with ideas of the Zone of Proximal Development (Vygotsky, 1978) and a tutor's assessment of where a student is and where they could be.

Much of the consolidation and some of the teaching on the Foundation Programme takes place through worksheets, all of which contain answers but few of which contain worked solutions. Students are encouraged to continually check their answers to ensure that they do not proceed to the next level unless they have mastered the pre-requisite sub-skills (Gagné, 1968, 1969, 1985). The worksheets are designed with a dual-purpose, not only to ensure that the requisite sub-skills are mastered but also to provide opportunities for students to demonstrate success, thus contributing to the cycle of positive reinforcement and improved self-efficacy Bandura (1994) mentioned previously. Additionally, regular checking might lead to improved self-monitoring and self-regulation skills (see Section on problem solving) and crucially prevent the development of incorrect associations and practices which would then need to be unlearned.

However, it has been noted (Section 2.1) that adults tend to take control of their own learning (Knowles, 1980) and consequently worksheet practice sometimes differs from the theoretical intentions. Adults that appear to be 'getting the right answer' for the first few questions sometimes elect to miss out the middle section and thus miss out on developing additional sub-skills that may be hidden within the questions. For example a series of equation questions may gradually introduce and combine skills with directed numbers or fractions. Additionally, they miss out on the opportunities for positive reinforcement.

Secondly, it has been noted that adults' expectations are often strongly related to their views of teaching from their past (Duffin and Simpson, 2000) and hence the practice of checking is sometimes linked with ideas of cheating. Some adults have been observed to deliberately cover up their answers. Positive reinforcement is sometimes embraced by students who have been observed to cover their own

work with a series of large ticks in red pen, despite the non-use of red pen in tutor marking practices. However, sadly, self-denigration in the form of large crosses has also been observed. The perception of marking as a 'success indicator' rather than a learning tool has also led to the practice by some students of only looking at answers after completion of the full worksheet, with the consequential risk of reinforcement of incorrect associations and actions.

A more fundamental issue with such worksheets arises from the assumptions that a correct answer implies competency in a particular sub-skill. Since the checking is with the answer and not the method, it may be possible that a student has not developed the requisite sub-skills intended by the worksheet author. Further, apparent success is unlikely to motivate students to search for alternative methods (Lithner, 2008).

An alternative form of assessment uses a portfolio made up of tasks completed within a student's own time and tasks completed under 'open book' test conditions. These weekly tasks and tests enable the opportunity to look at method as well as answer and allow the tutor to rapidly identify and rectify some of the issues that might be occurring. Although Gardner (1999) suggested that many tests are designed to show weaknesses rather than strengths, these tests are deliberately designed to maximise success and improve self-efficacy (Bandura, 1994) frequently matching wording and order with questions met the previous week on worksheets. As such, marks tend to be high but they have little discriminatory value (Clausen-May, 2001) suggesting that they would not be an appropriate test tool for this research.

The need for continual tutor interactions and assessment in order to inform decisions on the use of scaffolding (Wood *et al.*, 1976) was discussed in some

detail in Section 2.2. A key idea emerging from this discussion was the difficulty for tutors to identify a student's Zone of Proximal Development (Vygotsky, 1978) when student actions might not necessarily indicate a student was in the same position or even the same learning path as that envisaged by a tutor. In many ways the function of this process is as a matching exercise rather than a probing of understanding. Nonetheless, such interactions suggest that conversations between student and tutor have the potential to be developed further as a vehicle for investigating individual understandings within this research (see Section 3.2). However, if the research is to identify common behaviours amongst adults, it might be more appropriate to consider larger scale testing which is primarily associated with summative assessment practices.

### ***Pen and Paper Tests***

Larger scale testing tends to be used in public exams, entrance tests etc. and frequently involves the use of pen and paper tests. Much of the debate is on the validity of using performance in such tests to predict future performance in very different contexts which may require very different skills (Clausen-May, 2001; Gardner, 1999). Since the Foundation Programme aimed to prepare students specifically for future academic contexts, this could be argued to be less of an issue. More of a concern was producing a test to provide insights into understanding rather than simply generating answers. In public examinations, the need for clear and objective mark schemes tends to mitigate against the use of open-ended deep questions which might produce individual answers more complex to interpret (Clausen-May, 2001).

Since a part of this research involved comparisons with children, the need was to locate an existing test that focused on understanding. The CSMS (Concepts in

Secondary Mathematics and Science) survey (Hart, 1981a) from the late 1970s appeared to offer a set of high quality questions, all of which had been extensively trialled and tested for both validity and reliability. The CSMS survey is discussed in more detail within Section 3.3.

A concern with using questions for adults which had originally been designed for children was in interference of context. Pseudo real life problems, or 'school problems coated with a thin veneer of 'real world' association' (Maier, 1991, p.63), might potentially cause trouble if adults attempted to work to the real life goals inherent in the real life situation. For example: adults might be confused by unrealistic prices or have difficulty with questions requiring different length fish fingers (Hart, 1981a) when they know fish fingers come in standard sizes.

Developing a test is more than simply selecting questions. Whilst Clausen-May (2001) suggested that good design involved putting the more challenging questions at the end, this assumes that there is a known hierarchy of topics, which is a part of what is being investigated. Additionally, reduction of interference between questions requires deliberate mixing up.

Clausen-May (2001) also noted that anxiety might affect validity. A test might be measuring ability to control panic rather than ability to answer questions. This, together with the previous observations that the Foundation cohort might contain a number of people with high anxiety levels (Klinger, 2007) pointed to the need for careful presentation of the research tool. Throughout the research that follows and in all discussion with students the terminology, 'questionnaire' was adopted.

Before leaving discussion of pen and paper assessment, it needs to be emphasised that mathematical understanding must be far greater than that

indicated by such means (Black, 1999) or:

by focusing on the knowledge that resides within a single mind at a single moment, formal testing may distort, magnify, or grossly underestimate the contributions which an individual can make within a larger social setting.

(Gardner, 1999, p.100)

## **2.4 Hierarchies**

Mathematics is often referred to as hierarchical whether this refers to the subject content, the need for one concept to build on another (Skemp, 1979) or the way it is taught (Benn, 1997, Swan, 2006). There is certainly a frequent assumption of a direction of movement or development in learning (FitzSimons, 2007), as has been discussed in the previous sections. People move from lower to higher order skills, from simple to more complex schemata, from concrete to abstract, from standard problems to more complex ones.

Piaget suggested that children went through four stages of development, sensorimotor, pre-operational, concrete and formal operations (Piaget and Inhelder, 1958). Although later researchers have questioned the influence of context and language, and there is some debate over the exact age ranges and overlap, the broad notion of developmental stages is fundamental to many areas of mathematical learning.

Perhaps a subtle influence of the Piagetian legacy is the notion that one true hierarchy of mathematical skills exists or that groups of skills can be clearly placed in one stage or another of a hierarchy (Swan, 2006). Certainly, some skills require other groups of skills as prerequisites and patterns in mathematics sometimes



mean that improvements in one area can lead to improved performance in another without conscious effort.

Although there is no absolute hierarchy within concept development, certain competencies require many other competencies to have been previously mastered.

(Fillooy and Sutherland, 1996, p.141)

However, Fillooy and Sutherland distinguished between the need to develop some competences and concepts before it is possible to develop others and the notion of one 'absolute hierarchy'. Discussion in Section 2.1 has highlighted that skills improve with practice and proficiency. Different skills must develop unevenly in reflection of an uneven input. Thus, if skills were ranked in competence for one person this ranking might not be the same for another, particularly if they had undergone a different scheme of instruction emphasising different skill priorities. The use of data based on averages might also sometimes obscure this difference.

Gagné's (1968, 1969, 1985) notion that instruction should be related to the hierarchy of ordered skills required for a particular outcome was introduced in Section 2.1 and revisited in relation to Foundation Centre Practice in Section 2.3, but the discussion now needs to be taken a little further. Certainly, a teaching hierarchy might be developed from a logical sequence proposed by an experienced tutor, with an understanding of the whole work. There might even, on occasion, be some consensus amongst educators on *the best order*. Where there is difficulty, is when there is an assumption that this is the *only order*, yet Evans (2000) suggested that this myth is widely held.

If there was really just one possible hierarchy of sub-skills for each outcome, adult tutors would effectively be rebuilding the same building that had been built in

school. There would be less perception of conflict for adults between school and foundation teaching. Re-teaching would fill in the missing gaps, consolidate and reaffirm what has been remembered correctly and replace incorrect reconstructions with new correct information. Knowledge built elsewhere whether correct or flawed, whether potentially useful or not, might exist in a parallel storage system where it might or might not re-emerge in some alternative context. I suggest that this 'start again and fill in the gaps' approach is the underlying principle behind the design of many 'adult returner' resources.

However, there is not just one possible hierarchy. Gagné (1968, p.3) himself said:

Do they represent a sole learning route to the learning of the final task, or perhaps even a most efficient route? ... It is quite apparent that the answer is no.

An adult may have developed a set of alternative sub-skills through home and work. Starting again and filling the gaps fails to make use of this 'rich resource for learning' (Knowles, 1980, p.45) and denies opportunities for consolidation and empowerment. A more major problem occurs if the new building bears little relation to the old building at all (at least according to student perceptions). If a particular skill can be achieved by a number of different sets of sub-skills, successful demonstration of that skill only indicates mastery of one set of sub-skills, not all of the possible sets and not necessarily the set envisaged by a tutor.

A further problem with 'starting again' occurs if transformation of learning has taken place. In some learning, lower levels of learning simply act as pre-requisites for the next level (Gagné, 1985), so for example, working on questions like ' $3x + 2 = 2x - 5$ ' consolidates the skills required to solve  $= 12$ . However, if a lower level undergoes transformation and is subsumed to form a higher one, it is

not possible to go back to the previous level. In Piaget's theories, for example, it is not possible for a child to go back to an earlier stage of development although there may be some transition time when elements of both stages exist. Similarly, for schemata, if accommodation has permanently taken place, the old schemata cannot be revisited, although there may be an argument that sometimes accommodation is not fully embedded and, therefore, forgotten. More often this argument emerges in reverse when an explanation is sought as to why some accommodation appears to have been reversed and the answer is returned that 'accommodation cannot really have taken place'. As an example, if someone truly understands that multiplication does not always result in a bigger number, they should no longer be using this over-generalisation when solving problems. An alternative explanation for this continued use occurs using the theories of dual processing (Stanovitch and West, 2000) discussed in Section 2.5, which suggest that sometimes it is possible to give an instinctive 'common sense answer' (Coben, 2000) even though someone has the understanding to give a correct one.

This distinction between consolidation and transformation between levels is an important one for considering teaching approaches for adults. If consolidation occurs by revisiting earlier work, 'starting from the beginning' seems a possible strategy. However, if earlier work has been transformed and absorbed revisiting that earlier work may not be constructive or desirable. Two particular examples which highlight this dilemma are the use of automatic processes and the movement from concrete to abstract. The notion of automatic processes was introduced in discussion on skills memories (Section 2.1) and it was noted that there was some debate about whether it was appropriate to go back and break such processes into component parts to ensure understanding. The idea of

movement from concrete to abstract was introduced in discussions on schema but needs more consideration here.

The use of concrete examples within previous learning experiences might have influenced the selectivity of memory for adults, since credit, debit, scales, lifts etc. have all been met in real life and might be remembered or valued, whereas the abstract concepts derived from these concrete examples might have been forgotten. This might give rise to the idea that everything can be or should be derived from a concrete base and lead to a resistance to more abstract ideas or a, 'tendency to stay and progress within the concrete context' (Filloy and Sutherland, 1996, p.149). Whilst they argued it is possible to move backwards and forwards between concrete and abstract:

ultimately, what is sought is not the solution of a problem which the pupil already knows how to solve, but the way to solve a more abstract situation by means of more abstract operations.

(Filloy and Sutherland, 1996, p.149)

Skemp (1979) too argues of the need to detach a concept from its original set of examples to prevent them exerting a restrictive effect on future progress. Using these arguments together, the tendency for adult educators to start from the beginning and re-introduce a model that might successfully have been left behind could be considered detrimental.

### ***Testing the hierarchy,***

In order to measure the match between a proposed logical hierarchy and that found in students, one possibility was to divide the number of people who appeared to follow a hierarchy by the total number of people in the sample. A high value might well support the view that a particular hypothesised hierarchy was

similar to the learning hierarchy of most students, although Hart (1981c) suggests that a high value would also occur if topics were subordinate but not dependent. A low value implies that a hypothesised hierarchy is not being followed by the majority of the people in the sample. This does not necessarily mean that there is a 'better' hierarchy yet to be found but might imply that for this particular task, there are a number of alternative hierarchies in use.

In the CSMS study (Hart, 1981a) which is discussed in more detail in Section 3.3 children of different ages were tested and the facilities (percentage of correct answers) for each question were compared. A hierarchy of question types within and between topics was then produced using a complex system of grouping and association measures. Like Gagné's hierarchies, these hierarchies were set so that success at a higher level entailed success in all easier associated groups. Critics (see for example Swan, 2006) argue that most children would have been taught topics and skills in a similar order and a similar way leading to a similar pattern of knowledge and retention and that it is this order that is reflected in the hierarchies produced. In a sense, the work became a self-fulfilling prophecy.

If these same questions were to be given to adults, the results would demonstrate essentially a hierarchy of retention (or even of recall) rather than a hierarchy of understanding. Past teaching order may be less directly influential for adults, in that the differences in retention might be smaller for items learnt a long time ago (Gluck *et al.*, 2008). However, there is a counter argument that if something was introduced earlier, it might have been taught for longer and hence be more likely to be retained (Karsenty, 2002).

Failure to give the correct solution to a 'sub question' followed by success with the 'higher question' could be interpreted in multiple ways. The immediate surface

interpretation might be that someone has somehow achieved a higher order skill without mastering a lower skill – the lower skill is not necessary. The alternative surface interpretation is that someone made an error within a 'sub question', i.e. they did have a requisite sub-skill but had failed to demonstrate it, hence, the use of two out of three in the requirements for the CSMS study. A third interpretation is that a question may not be as valid for measuring skill achievement when used with adults as with children. For example, if all or nearly all children used a particular skill to answer a question, successful completion of that question could be said to demonstrate mastery of the skill. However, if a different group of participants sometimes used an alternative skill to answer a question, the validity of this question as a measure of the first skill is reduced.

Solving a mathematical problem of any kind requires appropriate strategies and decisions and these need to be considered when studying performance in tests. Further, mathematics understanding might be different for children and adults but so too might be the strategies for solving mathematics problems. Hence, discussion in the next section moves on to discuss problem solving in more detail.

## **2.5 Mathematical Problem Solving**

### ***Introduction***

It is apparent from the work so far that the learning and assessing of mathematics requires the 'doing' of mathematics, whether this is the completion of large numbers of very similar short questions, the working through of more synoptic but standardised problems or the attempt to solve much more complex, open-ended problems which may require generation of questions as well as solutions. All three types of activity are sometimes assigned the label, 'problem solving', although the skills used may be very different (Schoenfeld, 1992). Indeed there are sometimes

suggestions that the skills and beliefs acquired from working through large numbers of short, standard questions need to be 'unlearnt' for successful solving of more complex problems (Schoenfeld, 1992). Interpreting the problem solving behaviour of adults on the Foundation Programme will tend to focus on their actions with more standardised types of problems, although literature considering a variety of problem solving types is used to form a view.

Solving a mathematical problem is much more than simply mastering the requisite mathematical topics. A variety of other skills including problem recognition, selection of solution strategy and use of working memory are required (Glaser, 1999; Greeno *et al.*, 1999; Schoenfeld, 1992). Different writers have grouped problem solving requirements in different ways and with different priorities at different times. For example, comparisons of Schoenfeld's writings between 1985 and 1992 showed a growing recognition of the interaction of belief and affective issues (Schoenfeld, 1985, 1992). Following his review of problem solving literature Schoenfeld (1992) grouped problem solving skills as:

- The knowledge base
- Problem solving strategies
- Monitoring and control
- Beliefs and affects
- Practices.

The primary focus of much of the literature on problem solving is to consider the behaviour of experts, often in comparison to that of novices, in order to identify how to improve problem solving skills (Glaser, 1999; Schoenfeld, 1992). Certain key ideas have emerged from studies in specialist areas such as chess playing, radiography or even mathematics problem classes (Glaser, 1999; Greeno *et al.*,

1999, Schoenfeld, 1992) and studies of differences in behaviours between younger and older children (see for example Cowan, 2003). In general, novices are considered within a deficit model, as people who have not yet acquired the skills of the experts. However, Leron and Hazzan (1997) argued that analysing behaviour in this way was insufficient to explain all the actions of non-experts. Instead they argued that interactions with the affective domain, particularly if someone was experiencing confusion and loss of meaning, sometimes resulted in people making a series of decisions which they labelled 'coping'.

In attempting to consider behaviours through the eyes of the student, Leron and Hazzan (1997) suggested that coping activity involved trying to making sense of a problem, trying to meet the expectations of the person who had set it and using a 'principle of least effort' to decide if a solution was satisfactory. They illustrated these behaviours by describing the actions of students who tried to make sense by finding familiar words regardless of context and tried to meet the expectations of their teacher by hunting for something or indeed anything that they could say.

Although these descriptions resonate with some observations of Foundation student interactions, they are perhaps at the more extreme end of the spectrum, where the lack of knowledge base and perceived pressure produced particularly interesting responses. However, experts too need to make sense of the problem, identify the purpose and check that solutions are acceptable, (Glaser, 1999; Greeno *et al.*, 1999; Schoenfeld, 1992) even if their decisions are less likely to be influenced by the affective domain. Both experts and those heavily dependent on coping need to start with some form of knowledge base and both groups of people are influenced by their beliefs (Glaser, 1999; Greeno *et al.*, 1999; Leron and Hazzan, 1997; Schoenfeld, 1992).



Since Foundation adults are likely to be somewhere on the continuum between expert and novice, an appropriate framework with which to analyse their problem solving behaviour might have elements of both Schoenfeld 's (1992) and Leron and Hazzan's (1997) ideas. Thus, the sections that follow explore:

- The knowledge base
- Making Sense
- Meeting expectations
- Monitoring and control
- Beliefs.

### ***The Knowledge Base***

Mathematical information is much more than simply the declarative knowledge, 'knowing that', mentioned in discussions on memory. Nor is it just procedural knowledge, 'knowing how to' through the mastering of standard algorithms. The learner also needs to understand when this information can be applied, 'knowing why' or conceptual knowledge (Hiebert and Lefevre, 1986). Williams (1999) noted that school practices sometimes focused more on procedural than conceptual understanding (see also Baroody, 2003), highlighting the possibility that the adults for my research may have a variety of mathematical skills which they do not know when to use.

Before proceeding further, it needs to be noted that the words knowledge and understanding are frequently interchanged in writings in this area. Whilst some, like Baroody (2003), refer to both as knowledge, others refer to both as understanding and many, including Williams (1999) and Clausen-May (2001) refer to procedural knowledge and conceptual understanding subtly emphasising the difference between the two. Whilst I take a fairly pragmatic approach to the use

of these words in much of this study, there is perhaps an underlying view that understanding is more than knowledge. Hence my discussion on pre-knowledge, what is known before and interacts with what is to come, develops into a research question on understanding.

Schoenfeld (1992) defines the knowledge base to include not only relevant mathematical information (knowledge or understanding) but also consideration of how knowledge is accessed and used, or using cognitive ideas, how it is stored and retrieved. Earlier discussion on working memory (Baddeley, 1986) introduced the notion that working memory is a limited resource (Just and Carpenter, 1992) and certain forms of storage require less resource than others or aid 'cognitive economy' (Cowan, 2003). Greeno *et al.* (1999, p.136) noted that once skills are automated they can thereafter:

be performed with little cognitive effort, allowing attention to be given to more complex aspects of performance.

From latency studies it is suggested that children progress from the use of a variety of strategies for finding answers towards the more efficient and inflexible method of direct retrieval (e.g. Cowan, 2003; LeFevre *et al.*, 2003) which again helps with cognitive economy. Studies of expertise have shown that experts maximise storage through the recognition of meaningful patterns or the 'chunking' of information in a way that minimises long term memory space and allows rapid retrieval (Glaser, 1999; Greeno *et al.*, 1999; Schoenfeld, 1992). This is not simply number facts or arrangements of chess pieces but also observed in the retrieval of groups of equations (Glaser, 1999).

Use of information by an expert sometimes involves flexibility and choice even if this is subconscious. Cowan (2003) noted that older children were more likely

than younger children to select different methods for answering questions based on cognitive economy. However, LeFevre *et al.* (2003) indicate that adults appear to use multiple procedures and strategies and suggest that one of the reasons for this is the raising of confidence thresholds under testing conditions so that people are less likely to rely on recall. Although discussions on adult affective issues are considered elsewhere, Siegler's (1988) identification of perfectionist behaviour in children and the consequential wish to use procedures rather than just rely on memory, has real resonance with my observations of some Foundation students and has implications for consideration of research design.

Foundation students may not have the knowledge base of experts and consideration needs to be given to 'reservoirs of experience' (Knowles, 1980) which may contain incorrect information (see Section 2.1). Lack of use of number facts alongside competing associations from the world of home and work could have reduced the associative strength for correct answers (Cowan, 2003). Associations with other numbers and bonds, instinctive estimation, hidden counting and repeated recall and reuse of previous incorrect results can all lead to 'close-miss' errors (Cowan, 2003). Schoenfeld (1992, p.44) notes that:

Individuals bring misconceptions and misremembered facts to problem situations, and it is essential to understand that those *are* the tools they work with.

Thus a student with incorrect information will go on to use this as a resource for attempted problem solving. Evidence for misremembered number facts in my questionnaire responses might be near-misses or substitution of other results such as alternative multiplication bonds, although such errors could also be attributed to other causes.

## ***Making Sense***

When approaching a problem, experts usually spend time familiarising themselves with what is required and considering ways to proceed. In contrast, novices often identify surface features and move straight into calculations (Glaser, 1999; Schoenfeld, 1992). Novices try to 'make sense' by looking for clues to a known procedure or searching for, 'familiar faces in a crowd' (Leron and Hazzan, 1997, p.272). This can be a successful procedure, but Leron and Hazzan (2006) noted how words that stand out in the superficial reading of a mathematical question may send people in the wrong direction following 'red herrings' or cause them to abandon ways of working that might eventually have led to the correct solution. For example, Hart (1981b) noted that in a question requiring a percentage reduction some children appeared to focus on the word 'reduction' and went straight into a subtraction ignoring the word 'percentage'. Some of the issues around semantic influences are discussed in more detail in Section 2.6.

This search for sense and identification of surface features can be so rapid that people are not even explicitly aware that they are doing this. Instinctive or intuitive processes can be fast, unconscious and effortless and include the rapid recall of number facts (Cowan, 2003), pattern recognition (Glaser, 1999) and use of automatic processes (Greeno *et al.*, 1999), which have already been identified as part of the knowledge base. Coben (2000) suggested that adults have developed a set of intuitive processes that she referred to as 'commonsense mathematics'. The experience of adults in work and life situations, and this includes Foundation students, may well have reinforced the notion that these commonsense intuitions can be trusted, making them resistant to change (Fischbein, 1999). Unfortunately, instinctive processes can also lead people astray, causing them to respond to

'irrelevant external features of the task' (Stavy and Tirosh, 2000, p.85) or be influenced by the way in which numbers are introduced within a question (see discussion in Section 2.4). These sets of intuitive processes were referred to as System 1 (S1) processes by Stanovitch and West (2000) and this labelling has since been adopted by others (Leron and Hazzan, 2006; Kahneman, 2002).

Problem solving can also involve slower, more effortful, analytical processes, labelled as System 2 (S2) processes by Stanovitch and West (2000). As Leron and Hazzan (2006) pointed out, much of the focus of mathematics teaching is towards developing these analytical processes. However, even when people have developed advanced S2 knowledge, they do not always appear to use it when solving problems. Experiments involving undergraduate mathematics students (Kahneman, 2002) showed how people spontaneously gave an incorrect answer rather than calculate a correct answer. S1 processes could sometimes 'hijack' a problem if the question were written in such a way as to make some ideas more 'accessible' than others.

Fischbein (1999) also implied that it is possible to have parallel thought processes whereby the logical approach has overcome the obstacle but intuition pushes it back. Thus, for example, he describes that a college student might be very familiar with fraction calculations but still intuitively use the rule multiplication makes bigger (see Section 2.6). Going through the process of accommodation does not necessarily prevent intuition prevailing. The interaction of S1 and S2 processes is sometimes referred to as dual-processing (Leron and Hazzan, 2006; Stanovitch and West, 2000).

Leron and Hazzan (2006) argued for the need for students, as well as teachers, to have an awareness of such processes in their toolbox. Fischbein (1999) went

further and argued that knowledge about intuitive interpretations and the way these sometimes collide with taught mathematical ideas is crucial for teachers to understand the needs of their students. It seems clear that recognition of dual processing is at least as vital for interpretation of student behaviours within my study. If an adult provides an instinctive response to a question, this does not necessarily mean they are unable to also calculate something by alternative slower methods. If the instinctive answer is also incorrect, it cannot be assumed that someone does not understand an S2 process, only that they have elected not to use it. Making sense of the problem requires also an understanding of the purpose of the problem or an idea of a desirable outcome. Hence, the focus of the next section is to consider the need to meet expectations.

### ***Meeting expectations***

Leron and Hazzan (1997) identified that the need to meet the expectations of some 'authoritative other', usually a teacher or researcher, was a major part of the coping process. When this 'other' was present and the student was experiencing confusion and loss of meaning, the pressure to produce an answer and sometimes the ensuing panic might result in particularly wild answers. Even without panic, Lithner (2008) noted that students sometimes exhibited superficial behaviour which they believed was the acceptable response required by the teacher. Similarly, Skemp (1979) argued that students try to avoid their teacher's displeasure by giving verbal or written answers that they feel are desirable even if they have little understanding. Schoenfeld (1992) noted that sometimes in school mathematics, the problem and the teacher are inextricably linked; the teacher wrote the problem, the children used the method expected by the teacher and the teacher ratified the answer.

However, this need to meet expectations is not limited only to those physically present and might involve the teacher who was known to have set the questions or an unknown examiner. Setting students 'mocks', working through papers from previous years and reading examiners' reports and mark schemes are all embedded in good practice for A-level teachers. The purpose is not simply to test material or understanding but to help students understand examiners' expectations and to help students develop beliefs about how long or complicated an answer should be. Similarly, within the Foundation Programme, formative and summative assessment, examples of past papers and teaching itself are geared to meeting the expectations of 'academic practice'.

A number of researchers (see for example: Kirk and Ashcraft, 2001; LeFevre *et al.*, 2003) have identified that if the task shifts from simply finding a solution to explaining that solution, the need to meet expectations also shifts towards providing an acceptable explanation as well as an acceptable answer. Since the interpretation of the mathematics behaviour of Foundation adults in this study is likely to include adult self-reporting, whether concurrent with problem solving or retrospective, the findings of Kirk and Ashcraft (2001) and LeFevre *et al.*, (2003) need to be considered more closely.

Like Siegler (1988) and later LeFevre *et al.* (2003), Kirk and Ashcraft (2001) elected to set a series of simple number combination questions and measure the latency or time for participants to respond, although unlike Siegler their experiments used computer screens rather than cards. However, Kirk and Ashcraft's (2001) participants were undergraduate psychology students and the purpose of their study was to investigate whether behaviour might be influenced by instructions or rubric. The participants were split into four groups and given

different sets of instructions about providing explanations. Some significant differences in actions and reports of those actions were recorded between groups suggesting both issues of reactivity and veridicality.

Firstly, the awareness that a report on method would be required appeared to have caused some people to select different methods from the ones that they might normally use (reactivity). Secondly, there was evidence of participants trying to provide what they believed the researcher wanted. More people reported the use of direct recall in the group whose rubric more strongly emphasised direct recall and the converse was true for the group whose rubric more strongly emphasised use of procedures. Comparison with latency suggested that these reports did not always reflect methods actually used (veridicality) or perhaps people were using parallel methods and selected only to report the one they felt most appropriate. Since the distinction between direct recall, automatic and intuitive processes may be somewhat blurred, particularly for the people using them (LeFevre *et al.*, 2003), it was also possible that people were either unaware or had difficulty explaining instinctive methods.

LeFevre *et al.* (2003) suggest that the issues of veridicality and reliability observed by Kirk and Ashcraft (2001) may have been a product of the heavy biasing within instructions and not necessarily a problem with self-reports. Nonetheless, within my research, these issues need to be considered when interpreting written answers, for example if an answer and written method do not appear to match. As in Kirk and Ashcraft's (2001) work, it is within interviews that this phenomenon has the potential to be particularly influential. An awareness of this possibility needs to be integrated into the interview design and/or development of questions as the interview progresses.



### ***Self monitoring, Self regulation and Checking.***

Adults and novices alike appear to use various forms of checking during the process of problem solving, sometimes using, what Leron and Hazzan (1997) refer to as CAT (Clues, Answer, Test) cycles. Leron and Hazzan (1997) suggest that the major difference between these groups is in the number of cycles carried out. The confused student, reliant on coping procedures, may produce a verbal response and test it on the teacher (the authoritative other) to see if it meets expectations, searching for further clues from that teacher if it does not. If a problem appears to be solved satisfactorily there is little motivation to go further and find another way or understand more deeply (Lithner, 2008).

For the more advanced, the process of self-monitoring and self-regulation is in some ways testing against their own expectations and measuring progress against their own beliefs. Such monitoring guides decision making and is an essential part of expert problem solving (Glaser, 1999; Greeno *et al.*, 1999; Schoenfeld, 1992). For example, Schoenfeld (1992) described the way that experts working on non-routine problems in his problem solving class spent time pre-thinking, checked on progress and swapped to alternative approaches if required. In contrast, novices selected a route quickly and continued down the same path even if they were not achieving success.

Schoenfeld (1992) suggested that novices believe every question should be answered within 5 minutes. For an expert the timing may be better matched to task expectation and forms a part of self-monitoring. The recognition that sometimes too long has been spent with little progress may trigger the search for an alternative approach (Glaser, 1999; Greeno *et al.*, 1999; Schoenfeld, 1992). Conversely, fruitful lines of enquiry are not abandoned or assumed to be wrong

simply because they are time-consuming. Both of these decisions would be made within the context of the task; an exam question may have limited time and require the use of a restricted knowledge base, an open-ended question may require extensive periods of time for solution.

Benn (1997) noted that informal (real life) mathematics differs from school or formal mathematics in that people continue working in real life until a reasonable answer is obtained. Additionally, adult ownership of learning (Section 2.1) might encourage adults to persevere for longer than they might have done as children or be more willing to check answers so that they can improve. Benn (1997) suggested that adults use previous experience to check reasonableness.

In dual processing (Leron and Hazzan, 2006; Stanovitch and West, 2000), it was noted that instinctive, S1 processes can sometimes produce an incorrect solution even when the person answering the question has sufficient S2 knowledge to reach a correct solution. S2 processes can perform a monitoring and correcting role but only if they are triggered (Fischbein, 1999; Leron and Hazzan, 2006; Stanovitch and West, 2000). If adults are more likely to check their answers for reasonableness, this might have an impact on the use of S2 processes.

In conclusion then, if self-monitoring and self-regulation rely on comparisons with belief, the mixed experiences of adults may make it difficult to label their checking processes as expert, novice or indeed anywhere else on the path between. Consideration of such behaviours by Foundation adults may produce interesting results.

## ***Beliefs***

The last of the aspects of problem solving listed by Schoenfeld (1992) is that of belief. Interactions with belief have been encountered in previous sections but the remaining area to be considered is that of the interaction between belief and success. Studies of expertise (for example Greeno *et al.*, 1999) have suggested that success was related to the belief that you would succeed. The general connection between success and self-efficacy (Bandura, 1994), was introduced in Section 2.1 but some additional ideas emerge particularly in consideration of problem solving. The most important of these appears to be perseverance.

Greeno *et al.* (1999) suggest that perseverance is involved from the beginning, in sense making. If something is not immediately obvious, the belief that it will eventually make sense and, further, that the problem solver is capable of finding that sense, will cause someone to continue rather than give up. People who believe that 'many problems will yield to sustained effort will work on them longer, often with more success' (Greeno *et al.*, 1999, p.137).

Bandura (1994) suggested that self-efficacy improved through success and the resilience of this improvement was dependant on the difficulty of the problem and the perseverance required to overcome it. In effect improved self-efficacy was a reward for the determination to master a task rather than give up and the whole process potentially an upward spiral of success improving belief improving success. Indeed, Buxton (1981, p.16) observed this spiral occurring for adults within a problem, suggesting that as people got nearer to their goals, internal positive feedback 'appears to lend weight to our reasoning powers and enables us to drive through difficulties we might not otherwise overcome'. I suggested in Section 2.1 that adults' control of their own learning might increase the

determination to overcome obstacles and to persevere. Hence, this interaction of belief and success may be particularly marked for adults within my study,

This section on problem solving began with the premise that solving a mathematical problem was much more than simply mastering the requisite mathematical topics and has provided some valuable insights. However, clearly the mastering of mathematical topics is fundamental. Thus the final part of this review moves on to consider the mathematics itself.

## **2.6 The Mathematics**

### ***Introduction***

Writing so far has introduced ideas of situated mathematics (Lave, 1988), of mathematics in context of work or school (FitzSimons, 2005, 2007; Maier, 1991; Roth, 1999) and of mathematics in problem solving (Schoenfeld, 1985, 1992).

There has perhaps appeared to be an underlying assumption that the term 'mathematics' has a universal meaning, but this is not the case. Whilst a detailed discussion of the nature and history of mathematics may not directly inform this research, considering some of the alternative ways of looking at mathematics can generate a much richer picture of mathematics and its interactions with people and task. It has only been possible to represent a small part of that picture in the following discussion which focuses mainly on areas where there may be potential parallels with Foundation practices or where the vocabulary or labels introduced are likely to be of particular importance in future discussions.

### ***Alternative forms of mathematics***

The broadly anthropological study of the interaction of culture with mathematics, or 'the mathematics which is practised amongst identifiable groups' (D'Ambrosio,

1991, p.18), is sometimes referred to as Ethnomathematics. By identifiable groups D'Ambrosio does not mean just 'other' non-western cultures or national-tribal societies. His identification of cultural groups is much wider than this and includes urban and rural communities, children in certain age brackets, workers performing specific duties, professional classes and even groups of researchers (D'Ambrosio, 1994). This classification has some similarities with the notion of Communities of Practice (Lave and Wenger, 1999) introduced earlier.

Studies have taken place in different communities and cultures around the world. Gerdes (1996) provided a good overview of some of the work in this area. There are a growing number of examples of a quite complex use of mathematics amongst the illiterate and those with little schooling. The work of Carraher et.al. (1985) on the mathematics used by Brazilian street vendors is particularly well known. The descriptions of a child finding the cost of 10 coconuts by knowing the cost of three lots of 3 and adding one more (Carraher et.al., 1985) showed a strategy of repeated grouping and adding which were perfectly valid for success but were very different from the way such tasks might be carried out within a school situation. However, the strategies used of building and adding seem to be found throughout the world, not least during the tackling of ratio and percentage problems by children in British schools (Hart, 1981b). Examples of mathematics practice like those observed by Carraher *et al.* (1985) which have been shared within a community are sometimes labelled 'street mathematics' or 'practical mathematics' (Nunes *et al.*, 1993) but the label 'practical mathematics' is sometimes used as a broader label to simply distinguish between the mathematics practiced in school and the mathematics practiced outside (Evans, 2000). Adults have experience of both school and 'street' suggesting that setting a problem

which could be solved by either direct multiplication or by building and adding might produce some interesting results.

The designers of mathematics curricula often make certain assumptions about the understandings and beliefs of the people using them. Without shared beliefs on the concepts of time, money or what is to be valued, what is clear to one group is irrelevant to another and creates issues of culture clash or learning barriers when importing so called 'western mathematics' to other cultures (Bishop, 1991; D'Ambrosio, 1994). Less obvious, but perhaps more important for my own study, is D'Ambrosio's (1994) suggestion of culture clash between the mathematics learned in isolation by pre-school children, 'spontaneous mathematics', and the 'learned mathematics' expected within a school classroom.

He argued that this clash may force the earlier spontaneous mathematics to be lost and result in disempowerment for the child unless a way is found to incorporate the old culture within the new learning. This resonates with previous discussions in Section 2.2. Whilst Foundation adults are not pre-school, they will perhaps have developed some combination of spontaneous and street mathematics or Coben's (2000) common sense mathematics which might seem to clash with new learning. Extrapolating D'Ambrosio's (1994) argument suggests that adults too may be in danger of disempowerment.

### ***Consideration of mathematics topics for research***

A decision had to be made about which mathematics topics should be considered for study in this research. Since a major purpose of this research was to improve teaching to foundation students, the selection was narrowed to areas of mathematics which would be met again by all students whatever their eventual

degree route. Further selection required a somewhat iterative process starting from a large pool of potential topics and gradually eliminating them in response to further reading either on the mathematics itself or on adult mathematics use and beliefs. The work of Hart (1981a) and colleagues in the CSSM (Concepts in Secondary Mathematics and Science) research project provided a good starting point for consideration of many of the common mathematical themes. Fractions, algebra, place value, directed numbers or graphs, for example, are amongst those discussed.

The discussion that follows provides a fairly in depth consideration of the mathematics behind the topics finally selected for the main research, although a much wider range of reading was actually used in order to inform decisions about what to reject. For example, reading a selection of work on fractions helped identify that this topic might not be the most appropriate for this study. A more detailed explanation about why certain topics were selected rather than others is given in Section 3.3. The final selection of topics was: proportional reasoning, percentage use and the common over-generalisation that 'Multiplications makes bigger and Division make smaller' or MMBDMS.

Much mathematical research literature considers interaction with specific mathematical topics and reading such work leads to the development of some awareness of what is involved in understanding those topics. For the more in depth analysis of adult thinking on the three themes identified, a more systematic consideration of literature was required.

Clearly, it was not possible to survey all the literature on even one mathematical topic, the domain of research is vast. Instead the following descriptions provide summaries of some of the key ideas that keep reappearing whenever

proportionality, percentage or 'multiplication makes bigger' are encountered.

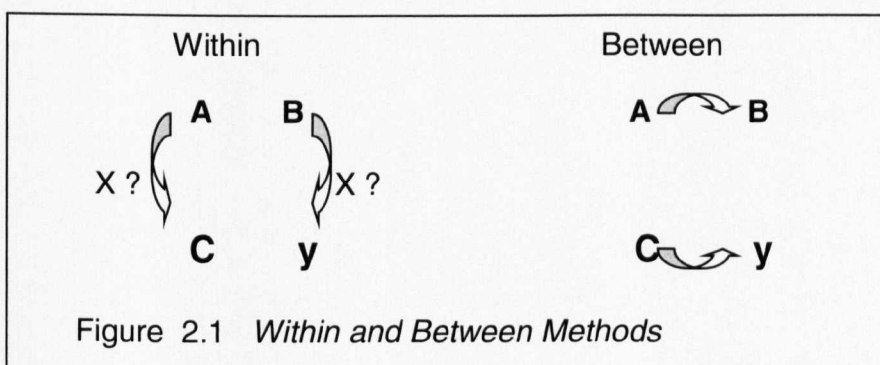
Many of these ideas are in common usage within the teaching profession and/or embedded within national curriculum strategies so that it is not always possible to attribute them to any one author or set of authors.

### Proportional Reasoning

Proportional reasoning could be argued to be one of the fundamental building blocks required for progress in some areas of science. There are many different types of proportional problems, including those involving geometry and those requiring the interconnection of complex information. However, the most useful for my work are the so called 'missing value' problems as illustrated in the typical question type, *If A requires B items, then C requires ? items.*

Different solution methods are described by different writers and grouped in different ways. Some methods may have evolved from others and the boundaries between them are blurred. Language and explanations differ, with some writers using notions of rate (Kaput and West, 1994) and others, like Lamon (1994), talking of composite units, but the methods described, if not the explanation for their use, are very similar. The discussion below indicates the approaches I have selected to use which seem to offer the most useful vehicles for my investigations.

A number of writers (for example Lamon, 1994) make the distinction between





methods which are 'within' and 'between'. These two ideas are modelled in Figure 2.1. A third method type, commonly referred to as the unitary method in British text books, involves finding the rate for one and then multiplying by a scalar.

Studies of methods used by children in the CSMS study (Hart, 1981a) noted that children changed method in response to perceived question demand, but that there was a predominance of 'within' methods of increasing sophistication, from the build-up methods, through to doubling, halving, trebling and multiplication by scalar. Some of the errors described in Hart's study are potentially functions of confusion between additive and multiplicative perspectives, argued by many writers to be linked to the transition from repeated addition to multiplication and by others to be a failure to identify when a problem is of a proportional type and when it is not (see Lannin *et al.* (2007) for further discussion on this latter case).

Lamon (1994) labelled the methods described so far as 'informal' because they were observed to be used by children before formal instruction took place. The methods of building and adding amongst Brazilian street children (Carragher *et al.*, 1985) also fit in this informal category. Formal taught methods might include the use of the algorithm  $\frac{a}{b} = \frac{c}{d}$ , but sometimes, the boundary between formal and informal is less clear. For example, Hart (1981b) observed children using the unitary method without a formal awareness that they were doing so. Part of the rationale for Kaput and West's (1994) work was the idea that informal methods outlast or heavily influence formally taught methods. Hart's observation of how few children remembered or used  $\frac{a}{b} = \frac{c}{d}$ , appeared to confirm this.

From their investigations into the proportional reasoning of adults at different literacy levels, Alatorre and Figueras (2005) confirmed that adults were often using informal methods when completing large numbers of questions in interview. They suggested that informal methods such as building and adding and particularly adaptive expertise were in use more with those less schooled, and noted that their one scientist, with a Ph.D in chemistry, behaved as a routine expert using the same basic strategy for all questions because he argued it to be the only method that was infallible. Alatorre and Figueras (2005) themselves pointed out that their sample was not representative and indeed they only considered 17 adults, but with some Foundation students being prepared for chemistry based routes their findings do add to the formal/informal discussion.

Studies of nurses completing drug calculations indicate a slight variation of this formal/informal usage. Pozzi *et al.* (1998) note that nursing students are taught the algorithm 'Amount you want divided by amount you have got multiplied by volume it is in', i.e. a version of  $\frac{a}{b} = \frac{c}{d}$ . However, they note that in practice, nurses spot patterns for particular drugs and use these informal calculations effectively. Indeed they suggest these methods may be more reliable and less prone to error under pressure, particularly important given the safety critical nature of these activities (Coben *et al.*, 2007).

Like Hart (1981a), Kaput and West (1994) also compared the percentage of children who were able to answer different types of proportion questions correctly but considered their results for individuals as well as for the whole cohort. They analysed the resulting hierarchies slightly differently from Hart, acknowledging that children were using an element of method selection, albeit instinctively. Kaput and

West (1994) attempted to draw out the features in questions which made them more amenable to particular solution strategies, distinguishing between numerical and semantic influences. Numerical influences included whether questions involved familiar multiples and/or gave the smallest ratio version making it easiest to build up. Semantic influences included whether questions encouraged rate thinking or the unitary method either in the way they were written or in their context. For example, a question on the price for a number of items might have encouraged thinking first of the price for one item.

The influence of number on method resonates with earlier discussions in problem solving on selection of method for cognitive efficiency (Cowan, 2003) but also fits with discussion on instinctive behaviour (Fischbein, 1999). A further theory, described by Greer (1994) as 'nonconservation of operation', suggests that even when two questions differ only in numbers used, some people are unable to recognise that the same operations can be used in each case. He argues that:

It has consistently been found that the numbers used drastically affect the ease with which the appropriate operation can be identified, even though, logically speaking, they have no bearing on it.

(Greer, 1994, p.69)

The influence of semantics seems to fit more with ideas of scaffolding (Wood et.al., 1976) where the language used helps lead people towards solutions or distracts them from the mathematics. Some people might be able to do the mathematics but not understand the vocabulary or grammatical structures used.

The notion that people use different methods for different numerical values and different contexts, and the existence of perceptions of nonconservation of operation, has far-reaching consequences for teaching on the Science Foundation

course where the numbers are often more complex and the situations less obviously encourage 'finding the amount for one'. It is not sufficient to assume that success in 'mathematics type' proportion questions is a basis for proportion problem solving in science, indeed this may explain many of the difficulties that students have with chemistry calculations. If my research indicates the predominance of informal methods amongst adult students, an appropriate response might be the refocusing of Foundation mathematics teaching onto the transition towards methods which are not dependent on specific relationships between numbers. Informal methods could perhaps be analysed and 'unpicked' so that they can be formalised and extrapolated or used to justify formal algorithms.

### Percentage use

From a mathematical point of view, percentage calculations can be seen as a form of proportional reasoning. Indeed, they appear in the ratio and proportion section of Hart's analysis (1981b). However, the recognition by a mathematical expert that operations are the same is not necessarily reflected by those answering percentage calculation questions. There is evidence in the CSMS study (Hart, 1981b) that children harness informal expertise in building, doubling etc., but I would suggest that the initial first step, the decision to find 10% or other starting value, might have been introduced through formal instruction. The use of the word percentage additionally separates it from proportion work in the eyes of a non-mathematician and links it to that body of knowledge stored under 'percentage'. For example, Hart describes the belief by some children that one always divides by 100 in a percentage calculation (Hart, 1981b). From a mathematical viewpoint, finding 1% could be argued to be linked to the unitary method, finding 10% links with ideas of composite units (Lamon, 1994 ) but short cuts like 50% rely on

number recall. Thus each question using percentage has the potential to be solved in a variety of ways and may offer a valuable opportunity to observe selection and interaction of methods in action.

The high profile perception of percentage use as a real life skill also sets it apart from proportion and causes some issues between different researchers. Dowling (1991) pointed out the danger of analysing the mathematics within a context from an outside mathematical view rather than within the context. He stated:

Successful transmission of images through percentages frequently demands little more than the ability to order, add and subtract simple numbers and, at most, a recognition that, for example, 35% can be represented by thirty-five out of every hundred. The ability to perform calculations with percentages is, arguably, never required in everyday life: most of us might be aware of the current mortgage rate, but probably have little idea of the extent of the impact on our payments of a 1% increase, and I have never met anyone who has felt the need to calculate repayments on any loan – we simply trust the lender.

(Dowling, 1991, p.96)

### Multiplication makes bigger

The existence of overgeneralisations in general has already been discussed in some detail in the end of Section 2.2. The particular misconception 'Multiplication makes bigger and Division makes smaller', referred to as MMBDMS by some writers, is of major importance because of its potential to block or inhibit future multiplicative problem solving (Harel *et al.*, 1994). Writers tend to focus on two areas: how this should be approached in the classroom and why this occurs on some occasions more than others.

Discussions on classroom approaches tend to consider ways to expose children to cognitive conflict and the success or otherwise of different activities (Swan, 2001). It has been noted (Section 2.2) that MMBDMS and other overgeneralisations, are often associated with the transition from integer to fraction and decimal calculation. Transfer from the belief that multiplication is always bigger to the recognition that sometimes it is not, is more complex than this simple statement implies. Children's responses indicate that decisions are often based on the actual numbers used in an instinctive way rather than some application of formal rule (Harel *et al.*, 1994). A conscious awareness of number interaction rules may conflict with instinctive, subconscious beliefs (Fischbein, 1999, Stanovitch and West, 2000) (see dual processing discussion, Section 2.5). Greer's (1994) suggestion of the non-conservation of operation' is also influential here in that people may make different decisions for two questions that appear mathematically the same. The MMBDMS statement may well be the merging of a series of sub-rules, belief that: multiplication can only be made by a whole number, multiplication makes bigger, division can only be made by a whole number, the dividend must be larger than the divisor, division makes smaller. Some of these rules might be dependent on models of division (Harel *et al.*, 1994). Harel *et al.* (1994) suggested that the differences in success for different sets of numbers, indicates the interaction of these rules. For example with  $0.2 \div 0.4$ , the rule division make smaller might dominate or the rule to swap the order to make the larger number first.

Harel *et al.* (1994), summarising the results from a number of studies on adults and children report a significant increase in difficulty from 'A x a positive integer' to 'A x decimal greater than 1' and a further increase in difficulty to 'A x a decimal less than one'. However, if the order was reversed, 'value x A' they recorded no

significant difference between questions with different A values. They argue that this was not so much a rule as a visualisation of counting sets of numbers, 'A' lots of whatever the value is, and suggested that children treated multipliers greater than 1 as if they were whole numbers.

## **2.7 Conclusions**

This chapter began by exploring some of the different theoretical approaches to learning. The last of these to be visited, the constructivist approach, considered learning as individual construction of knowledge and it was this approach that provided the overriding framework for this research. However, all four groups of learning approaches provided valuable contributions to this study, reflecting in part the complex interaction of influences within adult learning.

The behaviourist approach (Skinner, 1971; Thorndike, 1927) defined learning as observable change. Possibly the dominant model in much of education (Merriam *et al.*, 2007; Swan, 2006) as evidenced by university discourses on accountability and measureable outcomes, it was suggested that adult students themselves were sometimes pre-disposed to wanting this approach.

The cognitive approach centred on learning as a process within the mind. The emphasis here was on knowledge storage and how information was actively processed and linked to other information within a structure (Gluck *et al.*, 2008). Of particular relevance for this study was the discussion on working memory (Baddeley, 1986), on retention and retrieval in adults (Brase, 2002; Buxton, 1981; Karsenty, 2002; Kirk and Ashcraft, 2001) and on adult attempts to reconstruct knowledge to fill memory gaps (Duffin and Simpson, 2000; Karsenty, 2002).

The humanistic approach centred on belief in the human potential for growth

(Maslow, 1970; Rogers, 1983) and these principles, sometimes referred to as the 'Foundation Ethos', underpin Foundation Centre practices. It was recognised that mathematics could sometimes engender strong emotions (Bibby, 2002; Buxton, 1981) which could influence performance. Further, the Foundation cohort might contain a higher proportion of people anxious about mathematics than a standard undergraduate cohort (Klinger, 2007) and include people with negative self-image (Karsenty, 2004; Klinger, 2007) and low levels of self-efficacy (Bandura 1994). Considerations of androgogy (Knowles, 1980) highlighted that Foundation adults took ownership of their own learning (Benn, 1997), and had clear expectations of what teaching processes should be, possibly related to previous school experiences (Duffin and Simpson, 2000). From Schoenfeld (1992) came the notion that adults expect one right answer with Coben (2000) going further and suggesting that adults expect one right method.

Constructivist approaches were considered within three broad families. In the first of these families, construction was considered to be primarily an individual cognitive process with knowledge built through the development of schemata (Piaget and Inhelder, 1958; Skemp, 1979). Learning involved assimilating new information into an existing schema or recognising that it appeared to conflict. This 'cognitive conflict' caused a process of equilibration during which the schema itself might undergo change or accommodation. Some strongly held ideas might be resistant to change (Black, 1999; Skemp, 1979; Swan, 2006). Duffin and Simpson (2000) suggested that new information was sometimes classified as 'alien' by adults and hence avoided or ignored.

The social constructivist approach suggested that people make meaning through interactions with their environment, with peers and with more experienced others.



Vygotsky (1978) suggested that people are able to appropriate new learning during activities within the Zone of Proximal Development (ZPD). Developing from his ideas came the notion of learning in context and of 'situated mathematics' (FitzSimons, 2005, 2007; Lave, 1988; Maier, 1991; Roth, 1999). It was also highlighted that the major purpose of the Foundation Programme was to induct students into a Community of Academic Practice (Lave and Wenger, 1999).

The radical constructivist approach began from the premise that absolute mathematical truth was not knowable (orthogonal) (von Glasersfeld, 1995). Hence, people could only build their own constructions of knowledge and adjust them through testing for viability or 'fit'. The emphasis in this approach on trying to understand through the eyes of the learner and, particularly using Confrey's (1991) assertion that people proceed in ways that have internal consistency, appeared valuable for interpreting behaviour both for teaching and research.

All four groups of learning approaches emphasised the importance of previous learning. However, the adult 'reservoir of experience' (Knowles, 1980, p.45) might contain incorrect information (Schoenfeld, 1992), reconstructed knowledge (Duffin and Simpson, 2000; Karsenty, 2002) as well as affective elements (Evans, 2000) and it was recognised that all of these played a fundamental role in the development of new learning (Black, 1999). Considering the impact of memory and retention and the use of mathematics processes in a range of work and home contexts (FitzSimons, 2005, 2007; Lave, 1988; Maier, 1991; Roth, 1999), adult pre-knowledge had the potential to be much more diverse for a group of adults than for a group of children.

Gagné (1968, 1969, 1985) proposed that before new learning could take place a requisite set of sub-skills needed to be mastered. There was some discussion

about alternative hierarchies and the links between them, as well as the use of automotive processes and concrete examples.

Through discussion on overgeneralisations, but also following considerations of processes involving scaffolding (Wood *et al.*, 1976) it was identified that improvements of self-image and self-efficacy (Bandura, 1994) might be generated through structuring teaching processes to maximise experiences of success.

Further, changes to the students themselves through empowerment and ownership of learning (Benn, 1997; Knowles, 1980) might lead to a spiral of success through perseverance leading to more perseverance and more success.

The sections on problem solving focused more directly on the 'doing' of mathematics. Aspects of both Schoenfeld's (1992) and Leron and Hazzan's (1997) frameworks were used to consider problem solving behaviours. Problem solving began from a knowledge base which included methods of knowledge storage and retrieval. Proficient mathematicians exhibited a number of expert processes to store information efficiently for rapid recall (Glaser, 1999; Greeno *et al.*, 1999; Schoenfeld, 1992) aiding 'cognitive economy' (Cowan, 2003), although for some adults this storage could be patchy or flawed. In trying to 'make sense' of problems, novices were likely to be led astray by irrelevant surface features of a question (Glaser, 1999; Greeno *et al.*, 1999; Leron and Hazzan, 1997; Schoenfeld, 1992; Stavy and Tirosh, 2000 ). Some questions biased people towards intuitive processes and actions were so rapid that people were unaware they were using them, unable to describe their use (Kirk and Ashcraft, 2001) or using dual processing (Stanovitch and West, 2000).

Problem solvers tried to meet the expectations of an 'authoritative other', whether that person was present, past, or 'virtual' as in an imaginary arbiter of academic

practice. Experts carried out more checking than novices (Glaser, 1999; Greeno et al., 1999; Leron and Hazzan, 1997; Schoenfeld, 1992) and it was suggested that adults' searches for reasonableness (Benn, 1997) were more likely to lead them to such behaviour. Together, these problem solving behaviours showed that a student's answer might represent only a small part of their understanding.

Discussion in the Assessment Section identified that both conversations with individual students and pen and paper testing might offer useful insights for this research. The CSMS survey (Hart, 1981a) was identified as a suitable source for generating questions to be used in an assessment tool, which would be labelled as 'questionnaire' rather than 'test' in an attempt to reduce anxiety. The last section of the chapter considered mathematics itself and looked in detail at the three areas of proportion, percentage and the over-generalisation 'Multiplication Makes Bigger, Division makes Smaller' which appeared to be the most appropriate for use in this investigation.

Armed with the research questions, the theoretical background to both predict and interpret, and some preliminary ideas on assessment tools, the next section goes on to consider the methodology.

## **Chapter Three – Methodology – Design and Decisions**

### **3.1 Research Methodologies**

#### ***Introduction***

The previous chapters generated three research questions together with a theoretical rationale for their development. Additionally, certain key ideas emerged, particularly the wish to use empathic approaches and assume that students' own ideas have internal consistency (Confrey, 1991).

The next stage of the research process required some consideration of the range of methodologies available and, within them, the range of methods and tools that might be appropriate for this study. In selecting appropriate designs, it was noted that I was likely to have access to about 90 mature foundation students a year as well as some international ones. Equally importantly, it was noted that I did not have ready access to large numbers of children or schools and would need to rely on secondary data if needed.

A multitude of books on research methods were in existence, many of which appeared to contain some of the same material. For this reason, some of the discussion that follows was not attributed to any one source. However, three main sources (Bryman, 2008, Dawson, 2009, Open University, 2001) were consulted during preparation of the following sections.

#### ***Quantitative and Qualitative***

Methodologies were usually separated into two types, quantitative and qualitative. Quantitative methodologies tended to have developed from scientific ways of thinking with a strong focus on the creation and analysis of large sets of data that could be manipulated statistically. In contrast, qualitative methodologies tended to

focus on small scale in depth studies which could explore experiences, opinions or why people did what they did.

A brief review of the three research questions indicated the need for a mix of methodologies. A comparison of the pre-knowledge of Foundation Adults with children suggested an experimental design with a more quantitative approach, linking what was found with what theory suggested might be there. In contrast, identifying how people were thinking and why they might be making decisions suggested more intensive case studies of individual students might be appropriate and a more qualitative approach. Interaction of new and old learning was a more elusive concept. If learning was defined as change in behaviour, then a more longitudinal, before and after approach might be appropriate. If learning was defined as action, observation or reflection on 'doing' might be indicated. Additionally, the three research questions could not be considered to be independent from each other. How students 'doing mathematics' represented their understanding had implications for the comparison of performance in the first research questions and so on, Hence the findings from one area might well illuminate the work in another and mixed methods research offered more opportunities for completeness.

### ***Criteria for Assessing Research***

The Open University Research Methods in Education Handbook (2001) suggested that all research should be evaluated against just two criteria, Validity and Relevance, with Validity being based on Plausibility, Credibility and the Evidence which in turn was assessed for plausibility and credibility and so on. Plausibility involved identifying whether something appeared likely based on the knowledge available. Credibility involved a judgement about how believable something was

by looking at the likely threats involved in producing a claim and the size and direction of effect. Ultimately, this research would need to be assessed against this yardstick and these values would need to be considered at every decision point in the research process. However, ensuring that the evidence produced was both plausible and credible needed careful design of evidence collection methods. The detailed sets of criteria used for traditionally assessing research effectively provided models of good practice which were helpful.

### Validity

Traditionally, research and particularly quantitative research was required to demonstrate internal and external validity, reliability and replicability. Validity was the notion that something was broadly 'true' although it was the difficulty with 'truth' that had influenced the development of the Open University's (2001) alternative criteria. At the end of the research, validity might have referred to the integrity of the conclusions that had been generated (Bryman, 2008). At the beginning of the research, validity might have involved whether the questions asked or the indicators used truly measured or represented the concepts they purported to measure (concept or measurement validity). However, questions or instruments alone could not ensure validity. The method of administration needed to be such that differences between participants' responses were a reflection of the differences in that which was being studied and not caused by differences in external factors (external validity), in scientific terms there needed to be a fair test. Thus, there was an argument that for validity, a questionnaire for example, should ask the same questions in the same way and in the same order. There was of course, a counter model. In the ACACE Gallop survey (Sewell, 1981, p.7), the order of questions was deliberately reversed for half the interviews to reduce any

potential influence from people not answering later questions through 'mathematical fatigue'.

This requirement for standardisation of data collection highlighted an important issue for my own research design. It might well have been possible to design standardised data collection processes to allow comparison between foundation students. However, it was highly unlikely that secondary data would have been collected in an identical way. Nor would it have been sensible to try and redesign my own data collection process to mirror that used for children. The many differences between interactions with children and interactions with adults, and the probable differences between the purposes for which secondary data would originally have been collected and the purposes for which I would be collecting data, suggested that it was unlikely to be appropriate to match methods exactly. However, if information from two different sources were to be compared, these differences would need to inform the process of drawing conclusions.

An alternative measure of validity sometimes used was that of triangulation, where the findings from one method were used to corroborate the findings from another. For example paper records of people 'doing mathematics' could be compared with retrospective discussion about how they had done it.

### Generalisability and Sample Selection

It was often suggested that results were generalisable if the sample studied were truly representative of the parent population to which the results were being generalised. However, this presupposes that the important characteristics of the population that need to be mirrored in the sample were both identifiable and measureable. The ACACE Gallop survey (Sewell, 1981) for example used quotas based on gender, age, social class and occupation, stratified for the area in which

the studies were taking place. Since there may be many other unknown factors which were highly influential on mathematics performance, it is not known whether the Gallop samples were truly representative. Sewell (1981) herself thought it important to note terminal education age and mathematical education for her interviewees in addition to those Gallop criteria. Using these considerations in reverse, it needed to be identified what parent population the Foundation Programme cohort might represent and to whom results might be relevant.

The demographics of foundation programmes vary considerably between institutions. The programmes represented on the National Foundation Year Network recruited a mix of students who were international, recently failed A-Level, conversion (people changing subjects) or mature. For the majority, the core group was either international or failed A-Level students with mature returners constituting only a small proportion of their learners. The cohorts in this study were fairly unusual in that they provided a unique opportunity to study a larger body of mature students together (see Appendix 2a for more detailed information on cohort). There was no such thing as a 'typical mature student' or a national 'typical mature student cohort'. Therefore, there was no way of identifying the criteria on which a representative sample of 'typical mature students' could be selected from within my group of students. For example it would have been impossible to work with some form of stratified sample based on student age for my students, if the demographics of the mature student population were in continual flux. Further, any results from such selections might be seriously flawed if age were not the most important variable on which 'doing mathematics' depended. However, what this cohort had in common with nearly every cohort of mature students preparing for Higher Education across the country was the wish



to control their own learning and to succeed (Knowles, 1980)

The qualitative part of this research was likely to involve in depth studies of just a few students. The previous paragraph suggested that defining the criteria for such a sub-group was problematical. However, Dawson (2009, p.49) suggested that generalisability was 'not the goal' for many qualitative studies. She suggested that qualitative researchers believed that their work, 'might provide insights into the behaviour of the wider research population' (Dawson, 2009, p.49) but accepted that results might be different for another group of people because essentially no two people were the same. Bryman (2008) suggested that the use of purposive, rather than representative, sampling to select participants was often more appropriate for qualitative interviews and might be based on the relevance of particular groups for the research question.

Sewell's (1981) second set of interviews involved only 50 of the 107 people who took part in the first interview. She deliberately selected a higher proportion of women because they had shown more anxiety than men. She used responses to questions in the first interview about percentage to allocate people to three groups. Despite interviewing all those who she was able to contact and were prepared to be interviewed, her final selection involved a smaller proportion of people from the weakest group than the parent interview population. Since over half of those originally approached refused to take part in the first interviews and this may well have contained a larger proportion of weaker students, her final interview sample may have been even less representative of the wider population. Sewell (1981) did not claim otherwise. She described her final group as those who 'after the first interview and questionnaire, showed the most promise in revealing methods and patterns of mathematical competence, and were available for re-interview' (Sewell,

1981, p.40). Notice here a mix between deliberate selection of those most likely to help with her research questions and practicality, those who were available. Such compromises might well be necessary in my own decision making.

### Reliability

Another criterion used mainly for quantitative research was reliability, whether results were stable and likely to be the same on another occasion. The interaction of context (Lave, 1988; Maier, 1991; Roth, 1999) and of affective issues (for example: Bandura, 1994; Bibby, 2002; Evans, 2000; Leron and Hazzan, 1997) with performance, suggested that the results for some individuals might be more stable than for others. However, this did not immediately mean that performance would be different if the study were repeated, only that a repeat of the study would require a repeat of all the conditions under which the study took place, which might be problematical. Whatever the reliability of the behaviour of one individual, the reliability of the grouped behaviours of a sample is usually improved by increasing the size of the sample. Hence, it seemed sensible to maximise the numbers in this study and use a census rather than a subset of the whole foundation cohort.

Having decided on a mixed methodology, the next stage was to identify the most appropriate methods or tools to collect data. Whatever tools would eventually be selected, they were all likely to involve at some stage people 'doing mathematics'.

## **3.2 Research Tools**

### ***Introduction***

Section 2.6 had suggested that the mathematical areas for study should be related to those likely to be met on the Foundation Programme and produced a subset of mathematical topics. Section 2.3 on assessment had suggested that it may be

appropriate and justified for this research to use pen and paper methods but that methods used for formative assessment, asking questions and probing more deeply by asking for clarification would also be appropriate. The latter process relied on the interaction of tutor and student and the distinction between assessment and teaching was blurred. These two assessment methods seemed to broadly relate to two sets of research instruments: paper-based self completion questionnaires or their face-to-face equivalent structured interview schedules and semi-structured or unstructured interviews.

## ***Research tools***

### Self completion questionnaires or structured interview schedules

Self completion questionnaires, administered by post, email or in person, were frequently used in large scale surveys. If they were to be used for this research, they would need to be administered in person because the former methods would not provide standardisation of the conditions under which the questions were completed. Asking others or using books would invalidate any attempt at investigating pre-knowledge. Further, lack of confidence, high levels of anxiety and the perfectionist wishes of some adult returners (Bibby, 2002; Buxton, 1981; Coben, 2000; Evans, 2000; Karsenty, 2004; Klinger, 2007) might have predisposed people to using these strategies.

Typically, self-completion questionnaires contained mainly closed questions where responses needed to be selected from a limited number of possible options. Such responses were effectively pre-coded and could be rapidly inputted to data bases for processing. 'Doing mathematics' was generally an open activity and had the potential to generate an infinite number of alternative answers or methods. Attempting to pre-code or chunk options under 'other' in the questionnaire, risked

losing valuable information but it was not envisaged that post questionnaire coding of respondents' answers or methods would be problematical or cause difficulties with consistency. A possibility for this research, therefore, was to simply provide a set of mathematics questions within a paper-based questionnaire and ask people to 'do' them.

Structured interviews where interviewers followed a set interview schedule were sometimes used in surveys as an alternative to self-completion questionnaires, because they provided the opportunity to collect additional data through observation and to probe, albeit in a standardised, generic way. For example in the ACACE study, the interviewers read out a question from a card. Alongside the respondent's answer, the interviewer recorded the tools used for solution (pen and paper, calculator, mental calculation) and whether answers were given immediately or following a 'pause for thought' (Sewell, 1981). Additionally, interviewers noted whether a person was confident or unconfident. How the interviewers decided this, how valid their observations were and how high the level of inter-observer consistency was not mentioned in the Cockcroft (1982) report despite the conclusions given that certain groups were more confident than others.

Siegler (1988) and Kirk and Ashcraft (2001) both used standardised processes when studying use of number bonds, although the distinction between an experiment and standardised interview was a little blurred. Each person was shown a series of questions (on cards for Siegler and computer for Kirk and Ashcraft) and their answer and the time taken to reach it was recorded. Siegler (1988) also recorded any activity, such as finger counting, which might indicate methods being used. In Kirk and Ashcraft's (2001) study and in later versions of Siegler's (1988) study people were asked to describe how they had arrived at their

answer after they had given it. This idea of doing, observing and then asking to explain, appeared to have potential for my own research, but the actions, or more precisely the non-actions of an interviewer in this style of interview caused some dilemmas. Interviewing in this way would have been particularly problematical for my study when I had already established a learning/teaching partnership that did not fit this non-interactive mould. Instead, a semi-structured interview appeared to be more helpful and more akin to the assessment/teaching/learning dialogue highlighted as common Foundation practice (Section 2.2).

### Semi-structured Interviews

Like a structured interview, a semi-structured interview might have used some standard questions or had a list of the sort of areas that needed to be covered. Crucially though, this tool allowed the interviewer to interact and the flexibility to change the order, ask deeper questions, seek clarification or explore new areas as they emerged although the extent of this exploration might depend on the purpose of the research.

Such interviews were sometimes used in preparation for later more large-scale research, to identify appropriate themes, categories for closed questions or to trial previously developed questions and responses before wider use. Sewell (1982), for example, used a preliminary interview with 107 adults to talk about different situations in which adults used mathematics. She then used these findings to develop 'real life mathematics problems' which she administered in a follow-up interview to a subset of those originally interviewed. Unlike the structured Gallop ACACE research (Sewell, 1981) which followed on from her work, Sewell clearly noted interactions with interviewees and mentioned decisions to divert from questions that caused emotional reactions (ethical sensitivity).

Interviews could also be used to probe for deeper meanings and to clarify reasons for actions. Responses could sometimes provide triangulation for information from other processes, either for answers themselves or for researcher interpretations. For example, Siegler's (1988) use of children's retrospective explanations provided the opportunity for comparison with his own observations of methods used, although this assumed that the children's explanations had veridicality and could thus confirm Siegler's interpretations. Work from Fischbein (1999), Kirk and Ashcraft (2001) and LeFevre *et al.* (2003) for example suggested that if Foundation adults were to be asked to talk through 'doing mathematics' as they 'did it', their explanations might be limited, either by their ability to understand their own actions or to explain their reasoning.

### Unstructured Interviews

The previous examples focused on researcher led interviews, events where the agenda was set by the interviewer not the interviewee. Some qualitative researchers argued for the opposite approach, advocating a totally unstructured interview to prevent any possibility of imposition of a researcher's preconceptions, exploring where an interviewee wished to be rather than where an interviewer wished to go. Certainly, Bryman (2008) stated that attempting to understand from the interviewee's perspective, looking through their eyes was a main tenant of qualitative research. These ideas resonated with the radical constructivist approach to mathematics research discussed in Section 2.2, which emphasised the importance of truly listening, of being open to other views that have internal consistency but might differ from those of the researcher. Confrey (1991, p.121) suggested that although the interviewer selected the opening task, normally a non-routine problem with the potential for multiple interpretations, defining the problem

and deciding 'what constitutes an appropriate answer will evolve over the course of the interview'.

Interestingly, at one point Confrey (1991, p.133) referred to his interview as an 'extended teaching experiment'. As with Siegler (1988), the distinctions between an experiment and an interview were blurred. However, unlike Siegler's (1988) work which fitted well into a quantitative research classification, Confrey's (1991) interviews followed qualitative research processes.

### Selection of research Tools

The freedom to follow alternative thoughts and to value all ways of thinking within the radical constructivist approach (von Glaserfeld, 1984) was responsible for developing rich sources of information. However, this individualisation limited the opportunities for comparison. The ultimate purpose of this study was to improve adult teaching practices and Simon (1999, p.42) argued that development of an effective pedagogy required a focus first on what people had in common.

If each child is unique, and each requires a specific pedagogical approach appropriate to him or her and to no other, the construction of an all-embracing pedagogy becomes an impossibility.

Thus, the tool selected needed to be structured in such a way that common practices were more likely to be identified and effectively recorded. A small qualitative study comparing the understanding of a few students was possible but a larger scale quantitative approach was more likely to identify common issues.

A further concern was raised about the potential interaction of interviewer and interviewee. Whilst a self-completion questionnaire could be completed in private and anonymously, an interview situation required people to do mathematics in

front of others and could not be anonymous, at least to the interviewer.

Student desires to 'meet the expectations', (Leron and Hazzan, 1997) of a teacher by providing what they felt the teacher wanted (Kirk and Ashcraft, 2001; Schoenfeld, 1992; Skemp, 1997), were discussed in Section 2.5 and paralleled the 'social desirability effect' (Bryman, 2008, p.211) where interviewees tried to meet the expectations of the interviewer. This was of particular concern given my dual role as both researcher and tutor. My perceived power as a tutor with assessment responsibility made this particularly problematical and I had a concern that any discussion under these conditions would be so heavily influenced by these considerations that data produced would bear little relation to how someone was really thinking. This, together with ethical issues, such as lack of anonymity and raising anxiety levels meant that any interviews should take place at the end of the year when I was no longer their tutor. Later interviews could still provide the opportunity to look at questionnaire activity retrospectively as well as observing people doing mathematics and talking about it.

In summary then, following consideration of possible research tools available, the most appropriate tools for investigating 'adults doing mathematics' appeared to be self-completion questionnaires and semi-structured interviews. Both had the potential to provide valuable insights and provide complimentary data for triangulation and consideration of the research questions. Ethics considerations (Section 3.5) and concerns about the potential validity of data that might be collected in early interviews had resulted in a decision to delay interviews until after the completion of foundation teaching. Thus the research was likely to have two phases, the questionnaire phase when one or two self-completion questionnaires were administered to the whole cohort either before, or before and



after, teaching and an interview phase when a small sub-group of students participate in semi-structured interviews at the end of the year. The next stage of research design needed to consider the content of these tools in more detail.

### **3.3 Developing a Questionnaire**

#### ***Introduction***

The research questions included discussions of comparison and encompassed affective issues as well as mathematics itself, so a self-completion questionnaire or questionnaires needed to provide opportunities to investigate both elements. Further, the context in which such questionnaires should be given needed consideration in the light of some of the affective issues previously identified. Each of the four areas: mathematics content, affective element, overall design and administration needed to be considered. However, before doing so, it was helpful to identify some previous research that might provide guidance.

The work of Evans (2000) which focused on both attitude and understanding for a group of 1<sup>st</sup> year university students appeared particularly useful. He compared the performance of the adults reported in the Cockcroft (1982) report with the performances of his own students using similar questions. Evans adapted the ACACE questions (Sewell, 1981) to generate parallel questions based initially on either school mathematics or descriptions of more practical situations, to allow him to compare performances between different groups (gender, social class etc). He also considered the students' attitudes to mathematics. The major similarities between Evans' (2000) situation and my own suggested that Evans' methodology would be of particular value in the design and implementation of questionnaires. However, there were some major differences between our research aims and our two cohorts which would need to be revisited when decision making.

In particular, Evans (2000) used a cohort of 900 students. From the groups he selected, he classed over 60% of the students as mature (21 years and over). I had access to just 120 students per year, about 95 home and 25 international. Over 85% of the home students could be classed as mature (more in some years). Evans stated that up to half of the mature students, i.e. 30% did not have traditional entry qualifications i.e. 2 A-Levels. For the Foundation students probably about 90% did not have 2 A-Levels.

Possibly, the major difference between the two cohorts was the gap since studying formally. Not only did Evans have more students coming straight from school than the Foundation's 1%, but I suspected some of his non-traditional students had completed recent study in some form, whether this was through Access or part-time A-Level study. In contrast, many mature Foundation students were unlikely to have been in formal learning situations for some time.

Evans (2000) was considering numerate practices and selected mathematics questions which were primarily developed to test functional numeracy, albeit in school and 'practical' situations. I wanted to test deeper understanding. Further, he aimed to consider social difference, influences of gender and class and specifically set out to consider the influence of emotion in different contexts. I was aware that emotion would influence behaviour (Section 2.1) but this was not the primary focus of my research. Nor would it be possible within the limitations of my timescale to study separately many of the other potential influences on performance including gender, ethnicity, social background or school experience. Indirectly these influences would together be instrumental in the formation of the beliefs and behaviours of the individuals I was studying, but it would be difficult to attribute any differences in performance to one variable rather than another.

## ***Mathematics content***

Discussion in the previous chapter had already identified that the mathematical topics chosen for deeper study should be selected from amongst those required for foundation study. I elected not to study fractions directly. Although fractions were used on the foundation programme, the approach was different from that used within schools and heavily calculator based for most students (except future primary teachers).

Ratio and proportion were recognised to be fundamental building blocks for foundation study and identified in later reading as key concepts for mathematical development. Percentages were known to be of major importance for foundation study and had the added value of being heavily associated with life skill, increasing the likelihood of interaction between different learning. Sewell (1981), for example, emphasised their importance on the numeracy agenda.

As a deliberate contrast, algebra was chosen in the original list because it was unlikely to have been consolidated in adult life. Some interesting findings emerged from the algebra responses in the 2006 study, particularly the discovery that some students could cope with quite advanced algebra yet fail on basic calculations (Dodd, 2008). This topic looked promising for future study, but a fully informed analysis would have required a major investment in research time and I believed this to be of less value to my research questions than other areas.

Students had also spent a disproportionate amount of time on these questions. As a compromise, algebra questions were moved into a second 'extension section' in later questionnaires. Similarly, questions on directed numbers, which seemed to generate little research value, appeared only in the pilot. A final mathematical topic was chosen to focus more on interaction with belief. This was the notion that

'Multiplication makes bigger and Division makes smaller' discussed in Section 2.6.

Both Hart (1981a), discussed in the next section, and Evans (2000) elected not to allow calculators, although for different reasons. I decided to follow their example for the 2006 study but later allowed calculator use in the new extension section.

Since at least a part of this research required comparison with children and access to groups of children for testing would be problematical, the first stage required consideration of information publically available on both questions and results.

A wealth of questions and results were available from public exam boards but were of limited value for my purposes. Practical considerations mean it is not possible to test every aspect of every topic in every context in one exam, so at best exam results could only have provided a snapshot of competence. Further, mathematics questions are often synoptic, and require the integration of a number of skills including problem recognition for success. Thus, a person with a good understanding of percentage use might be unable to demonstrate this because they were unable to access the part of a question where it would be required. Indeed, Gardner (1999) has suggested that some assessments are deliberately designed to identify weaknesses rather than strengths. The use of more open ended questions which might explore deeper understanding is generally prohibited in public exams by the difficulties in providing consistent marking.

Few tests and surveys attempted to map the complete understanding of a mathematical area. Answers to a few questions were considered as representative of understanding of the whole topic. A major exception is the CSSM (Concepts in Secondary Mathematics and Science research project) survey (Hart, 1981a) which was mentioned in Section 2.4. This survey was carried out in

the late 1970s and involved extensive interviewing and/or testing of about 10,000 children on eleven different topic areas. The book, 'Children's Understanding of Mathematics: 11-16' edited by Hart in 1981 summarised the information from the study and includes about 50% of the questions used. Not only were the questions extensively trialled and tested to ensure validity but the resulting analysis produced real insights into the different ways children understood mathematics and how and why they approached problems in particular ways. Using similar questions for Foundation adults would provide some opportunities for comparison of both the percentage of correct answers (facilities) for each question and the methods used and at the same time benefit from the question validity. Additionally, since Hart and her colleagues identified hierarchies within and between topics by comparing question facilities, use of similar questions for adults might also allow such comparisons.

In the CSMS study (Hart, 1981a) a number of questions on the same topic were given to a group of children. Some of these questions might be considered to be mathematically equivalent in that they tested the same or nearly the same concept, but differed in context, number and semantics. Levels were allocated to children based on successfully completing two thirds of the questions at that level, so the repetition of questions on similar concepts allowed greater reliability in this allocation, reducing the influence of so-called 'random' errors. In contrast, for this research with Foundation students, the main emphasis was not on allocating the level of an individual in one topic but on investigating the 'confused mental map' of adults (Coben, 2000) and identifying whether their patchwork of knowledge might be distributed unevenly between topics. For this reason, adults would need to be given questions on the five topic areas identified, not just on one.

It was not realistic to expect students to undertake five sets of testing on separate occasions. There were two options; to test the whole cohort of students on a reduced set of questions on all topics or to split the cohort into five subgroups and test all the questions on one topic using a reduced number of participants. The former appeared more reliable since it maximised the number of people answering a particular question. Reducing the number of questions on one topic reduced the opportunities for repeat testing of concepts. However, it had earlier been suggested that the belief in one correct method was common amongst adults (Coben, 2000). If an adult selected one method and proceeded to use this method for all 'similar' questions, the value of repetition for reliability was reduced since answers to questions could not be considered to be independent of each other. Using fewer questions but interspersing them with questions on other topic areas might be more appropriate. However, such a decision might have implications when comparing results for Foundation Adults with CSMS students.

A decision not to simply use every question from the CSMS survey necessitated a further decision on which questions to select, reject or adjust. An additional influence on this decision was the potential interaction of mathematics from everyday life. Hence, the set of ten questions focusing on this area which were used in the ACACE adult population survey completed by Gallop (Sewell, 1982) were also of interest. An extract from the notes used for question selection is given in Appendix 1f.

### ***Affective Issues***

There were a number of different affective issues to be considered which might influence the research in different ways. The first of these was anxiety which could relate to mathematics as a whole or to specific topics or activities (Bibby,

2002; Buxton, 1981). A particular concern was to monitor, at least initially, whether the questionnaire might be causing exceptional levels of anxiety. Additionally it might have been helpful to identify whether general anxiety about mathematics had the potential to effect activity.

Likert attitude scales were increasingly popular as an indicator for the concept of anxiety. The Likert method normally required a respondent to indicate their level of agreement with a statement, from strongly agree and agree at one end to disagree or strongly disagree at the other and with neither agree nor disagree in the middle. Variations used more points on the scale, some using words, some using numbers and most using both. Whilst an absolute score for one answer had little value on its own, the use of such scales enabled comparisons between statements and between respondents. The principle behind using such tools to investigate anxiety was not to compare students whose notions of rating may be different but to provide some alternative activities or events that acted as a baseline for comparison. There were a number of variations of this model in existence, but the template used by Evans (2000) to investigate anxiety in his first year undergraduates appeared particularly useful, since the activities he had chosen as baselines looked similar to the likely activities of my own undergraduate students. I adapted Evans' template and asked participants to rate the anxiety level for 24 activities, including, 'completing this questionnaire' (see Appendix 1c).

The research questions involved consideration of pre-knowledge including belief and it has been suggested that firmly held beliefs may be difficult to change (Skemp, 1979; Swan, 2006). Some indication of belief, confidence or trust in particular answers would be valuable, if there was a possibility that interaction with these emotions would affect behaviours. Measuring such concepts was

problematical. It has already been mentioned that in the ACACE Gallop survey (Sewell, 1982), interviewers recorded someone as confident or not confident based on the way in which they answered the question but this seemed somewhat subjective, even assuming that outward behaviour mirrored inner feelings.

The simplest tool to use when investigating belief might have been the question 'Do you trust your answer?' asked after each mathematics problem. However, yes/no answers are generally of very limited use. Similarly, the two options, confident or not confident used in the ACACE study would have given insufficient indication of the level of confidence. Use of a Likert scale was problematic. Not only was it poor practice (Bryman, 2008) to use Likert scales to measure two different concepts, anxiety and then trust in the same event, but Likert methods typically measured items in relation to each other. For example someone who might have allocated an anxiety value of 3 for attending a mathematics class and a 5 for attending an English class, might have decided to allocate a 4 for attending a science class because they identified it as not quite as bad as mathematics but not quite as good as English. In contrast, the focus when investigating trust was not whether one answer was trusted more than another but whether the answer was trusted or not. Further, since each problem would need to have been completed before a trust level could be allocated to the answer, internal consistency might have been reduced by the interruption in thought. Labelling appropriate points on a scale could also have been problematical. The middle position of neither trusting nor distrusting an answer did not match a real belief.

Whatever the form of the additional question selected, it also had to serve two additional purposes. Discussion on problem solving in particular had identified that methods were selected and subject to change in different contexts. An



indication that someone had little or no trust in their answer suggested that a different solution might be given on another occasion. In contrast an indication of a high level of trust suggested an alternative solution was less likely following Lithner's (2008) assertion that if a satisfactory solution is found there is little motivation to search for alternative or deeper meanings. Thus the additional question had the potential to indicate the replicability or stability of problem solutions and hence the measurement validity.

The second additional purpose was to indicate whether it might be appropriate to allocate resources to interpreting alternative solutions. A confident answer suggested that some form of rule or consistent method was in use and worth investigation. However, there was the possibility that a guess was based on someone's lucky number or on a letter in a multiple choice section that 'hadn't been used yet'. Identifying such an answer as a guess might have indicated that less time should be spent searching for logical explanations.

One possibility would have been to provide a filter question. Is your answer a guess? YES/NO. If your answer is not a guess how confident are you? However, Bryman (2008) suggested that filter questions should be avoided in questionnaires when possible because of the possibility of confusion and non-response or non-engagement with later questions. Additionally, there were two kinds of guesses, those based on some intuitive or instinctive reasoning (Fischbein, 1999) and those based on factors external to the problem as in selection of a lucky number. The former might have had the possibility of being correct or at least providing some insights. The latter was unlikely to provide value. Providing the option of an unlikely answer alongside the option of a guess made this distinction more explicit.

Putting these ideas together, the tool chosen to provide additional information about answers needed to be a set of options rather than a Likert scale. These options needed to provide clear and meaningful descriptors so that responses had validity and could be independently related to each answer rather than dependant on relative comparison between answers. The options needed to be a closed set which were mutually exclusive but sufficient to capture the full range of trust. Whilst 'Just guessed' was an action rather than a feeling, this would also be offered to allow for the occasions when there was no trust whatever in the answer. A good set of options required to be balanced, so that an equal number of options were linked to ideas that a given answer was correct and to ideas that a given answer was incorrect. A middle option indicating that someone believed an answer neither correct nor incorrect did not represent a realistic stance in the light of the right or wrong mentality of adults already discussed and so would not be offered. These deliberations suggested that four options should be available. Using Likert type language, the two options which suggested a respondent believed an answer was correct could be 'very confident' and 'confident'. However, this labelling became confusing if used to describe answers believed to be incorrect. 'Not confident' and 'Very not confident' are cumbersome and reordering the wording to 'Not very confident' and 'Just a guess' had more real meaning. It was the meaning that had priority not the equality of categories.

A well-designed questionnaire needed to be clear and easy to understand with some consistency in layout. Following each question or each part of a question with a further question on confidence had the potential to create a bulky and confusing questionnaire and would result in considerable repetition of the same instructions. An alternative layout was required. It was a common practice in a

number of subjects on the Foundation Programme to ask people to link confidence ratings (confident/not confident) to topics. This information might have been used to separate people into groups or identify the most appropriate areas for a workshop or revision session. The usual method for such processes was to provide information in a table format with two columns to the right for selection. This format had also been used successfully in preparing for mathematics revision sessions, when people have been asked to look at questions rather than topics, although people were usually identifying whether they believed they could do something rather than actually doing it and evaluating the quality of their answer. It, therefore, seemed appropriate to trial a similar format for the questionnaire.

### ***Personal Information***

Evans (2000) required a variety of demographic data from his participants in order to compare performance of different groups. Whilst demographic data might have provided additional background for my research it was not an essential part of it and there was a possibility that collection of such data might be seen as invasion of privacy or might compromise anonymity. From a practical point of view, unnecessary questions might have led to questionnaire or later mathematics fatigue (Sewell, 1981). I, therefore, limited the information requested to when mathematics was last studied, highest mathematics qualification and progression route.

I felt that the time spent since leaving a formal learning environment might be an important influence on performance and, hence, a question should be included on how long ago mathematics had been studied. Evans (2000, p.33) had felt that a question on age would act as a 'proxy ... or substitute for measures of experience in practical, out-of-school activities requiring numerate thinking' and could allow

any such effects to be controlled for. However, previous discussion had suggested that this influence would be more dependent on what those out-of-school activities were, rather than how long they had been practiced. Further, consideration of retention and memory had suggested that there might be a levelling off in effect after a few years (Gluck *et al.*, 2008). The rubric was changed slightly after the 2006 run to reduce ambiguity and prevent trivial answers such as 'at school' to 'When was mathematics last studied?'.

Sewell (1981) included mathematics qualifications in her interview sample data and I thought that the qualification studied might be influential in the type and level of mathematics covered and hence student performance in the questionnaire. It seemed of potential interest to identify whether certain misunderstandings had carried through even to those who studied mathematics at higher levels. Evans (2000) advocated the use of qualification level as a replacement for participation measure but identified it particularly as a variable to be controlled for. At this stage it seemed sensible to identify what qualifications level people had and delay a decision about how to use this information until later.

### ***Administration Concerns***

Evans (2000) had used an initial questionnaire with three parts: an experience scale with general questions about the students and their experiences of mathematics, a performance scale with actual mathematics questions similar to those used for the ACACE study (Sewell, 1981) and a Situational Attitude scale which had asked students to rate their anxiety level for different tasks. Experience had indicated that a few of my mature students might have very high levels of anxiety, akin to Buxton's (1981) panic. Entry to the first year of a degree usually required some form of minimum qualification but Foundation entry did not require

formal qualifications. Potentially, the Foundation cohort could include some weaker students and the academic make-up of the cohort could vary considerably from year to year. At the other end, some of these foundation students were taking advanced mathematics modules so might find basic questions too trivial (see Appendix 2a for Foundation student information).

Despite these differences, I decided that Evans' questionnaire format could still be considered a useful template for the first run. I decided to use a similar pattern for the 2006 cohort with three sections: personal information, attitudes and mathematics knowledge

The attitude section which was designed to measure anxiety was based on the template of Evan (2000) as discussed previously. The results for the 2006 cohort were found to be unreliable and of less value than other directions of enquiry. (The discussion on responses from the Year 1 Final Report (Dodd, 2007a) has been copied to Appendix 4 for reference.) Given these issues, I decided not to include an anxiety rating section in future questionnaires, but did replace it with two simpler questions on anxiety about study in general and study of mathematics in particular on the Foundation programme. I hoped that the inclusion of such questions would provide a non-threatening start to the survey and additionally emphasise the distinction between a test and a survey.

Evans was concerned about minimising pressure and anxiety and elected to administer his questions in other people's classes to distance the survey from any formal mathematics assessment. Given some vital differences between my own cohort and that of Evans, I chose the opposite approach. Whilst the majority of students in a typical 1<sup>st</sup> year cohort were progressing from a previous education establishment, the majority of students on the Foundation programme were

mature students who had not been in formal learning situations for some time. Alongside severe anxiety about mathematics, students might have feelings of insecurity and deep concerns about failing. Few would have had recent experience in test situations or in reading rubric. These problems were compounded by the need to administer the survey early in teaching. Since experienced foundation mathematics tutors were particularly skilled in dealing with such anxieties and had strategies to reduce them, they seemed the most appropriate people to administer the questionnaire.

We have an holistic approach to teaching and as Strand Leader it was not possible to distance myself from students. Instead, students needed to be convinced that the survey would not and could not be used for assessment or judgement. Any form of anonymous number use required trust in the system but students would not have had time to develop this. It was decided to identify scripts by primary school. This was a transparent system enabling follow-up questionnaires to be matched but preventing direct identification without the aid of the students themselves. In later questionnaires a second identifier, 'mother's first name' was included to improve the matching process (see Section 4.1).

I believed it unreasonable to expect students to risk possible anxiety only for my research or for the benefit of future students. A commitment was given that, early in the research, scripts would be analysed by group and information given to tutors so that teaching of current students could be enhanced.

Evans limited the time for each section to prevent people revisiting questions. It was decided to limit the time to 30 minutes for the mathematics questions and 10 minutes for the attitude section in my survey to provide a balance between prolonging anxiety and allowing time to gain confidence. The content of these

sections would be adjusted to fit these times. In 2006, I had been aware that limiting time might add additional pressure for some students. To build up confidence for exams, I already used untimed weekly tests as part of the mathematics course but had not really formalised my reasoning behind this. Further reading about the implications of restricting time for some students (Walen and Williams, 2002), increased my concern about this issue. Additionally, when the papers from the 2006 study were analysed, I realised that the limited time and instruction not to look back might have been partially responsible for a number of gaps left by some students which caused difficulties with analysis.

In response to these issues, mathematics questions were split into two booklets in later questionnaires and students encouraged to complete all the questions in the first booklet, revisiting gaps if necessary, before looking at the second. I hoped that this would increase the probability that any blanks represented an inability to answer a question, rather than a strategic decision to spend the limited time on questions which were more easily recognised. The second booklet, which allowed calculator use and contained a few calculations and some algebra, was identified as of lower priority and students were asked to only proceed to this if they had done as much as they could of the first booklet. No pre-limit was placed on time, but the process was stopped when the last student finished the first booklet.

### **3.4 Preparation for interview**

#### ***Introduction***

It had been identified that a semi structured interview was the most appropriate form of interview to use and had the major advantage of flexibility. The decision to conduct interviews after the end of the year provided the opportunity for some preliminary questionnaire analysis to take place prior to the event. This analysis,

alongside the research questions themselves provided the reasoning for the inclusion and prioritising of certain elements.

### ***Interview Questions***

The three research areas under investigation involved comparisons of understanding with children, considerations of behaviours and reasons for those behaviours and consideration of the interaction of old and new knowledge. The interviews had the potential to generate valuable insights in all three areas.

It was apparent that a major part of the interview needed to involve people 'doing mathematics' and provide the opportunity for me to observe their actions, listen to their explanations and probe more deeply for their reasons. Consideration of the interaction of old and new knowledge, perhaps needed to identify as much what people knew but rejected, as what they had actually selected to use.

Additionally, since responses within questionnaires were sometimes open to multiple interpretations, retrospective explanations from the students themselves might provide elements of clarification or triangulation. However, it was also possible that students revisiting questionnaire scripts might use elements of reconstruction (Duffin and Simpson, 2000; Karsenty, 2002) to fill in gaps either when they could not remember what they had actually done or were unable to explain instinctive methods (Kirk and Ashcraft, 2001; LeFevre *et al.*, 2003).

It was decided to ask students to work through a series of mathematics questions first and then to consider their script responses together afterwards. Preliminary questionnaire analysis had indicated that number might have been more influential than expected in method choice and a deliberate decision was made to return, where possible, to the numbers used in the CSMS study (Hart, 1981a). Two extra



questions asking for  $\frac{2}{3}$  of 12 and  $\frac{3}{4}$  of 12 were added to investigate further the phenomenon of 'swapping' which was emerging from some questionnaires.

Whilst interaction of old and new knowledge might be indicated at various points within the interview or emerge through deeper questioning about memories of other methods, it was decided to also ask students directly whether they had ever been aware of any conflict between the new teaching and their old knowledge.

### ***Preparation for Recording and Analysis***

In order to be able to focus fully on the interview, it was intended to use audio recording if interviewees were in agreement. This seemed less intrusive than the use of video. Although transcription was recognised to be a very time consuming process, it was known that full transcripts were preferential and would be practical since my research involved only a small number of interviews.

Whilst the coding and analysis of questionnaires did not look problematical, the future analysis of interview transcripts seemed less clear. Decisions needed to be made about the processing of responses. Two approaches seemed common. One, based on 'Grounded Theory' (Strauss and Corbin, 1998) started from the transcript itself, analysing small sections or even line by line and allocating provisional labels or codes. These were then grouped together or renamed as concepts appeared to emerge from the data. The other common approach, thematic analysis, involved collecting information under themes as its label suggested. However, this broad label really encompassed a range of very different approaches. Themes could evolve entirely from the data, from topics or ideas that appeared to be repeated a number of times, or be developed in advance from theory or research questions. Sometimes themes used could be a

mix of both. Following identification of themes, related dialogue, or snippets, could be extracted from transcripts and pasted into some form of accessible chart. The Framework matrix (Bryman, 2008) which stored abbreviated versions of material in cells according to themes, sub themes and interviewee, appeared particularly helpful. A full description of the analysis actually used is given in Section 4.2.

### **3.5 Ethics Considerations**

#### ***Introduction***

Whilst ethics considerations are an essential part of all research planning, many of the underlying principles of ethics were encompassed in what I have labelled the Foundation Programme ethos, the commitment to broadly humanistic principles, student ownership of learning, learning for empowerment and mutual respect (see Section 2). These principles were already firmly embedded in my practice and underpinned many of the decisions made for this research.

Two particular aspects of this research needed special consideration. The first was my dual role as both tutor and researcher. The second was the potential emotional vulnerability of some of the students. This vulnerability did not just relate to those with severe anxiety about mathematics (Bibby, 2002; Buxton, 1981) but also recognised the uncertainty, insecurity and self-doubt of those who might have made a major life commitment in returning to learn as an adult. Indeed, there have been a number of occasions in this research when my definition of harm has been found to be much wider than the definitions of those considering my research on an ethics committee. Beyond ethics and the Foundation ethos, failing to recognise the implications of actions which affect emotion had the potential to seriously compromise the validity of this research.

Widening the definition of harm still further to include depriving students from an opportunity to learn, meant that decisions could only be taken to replace a teaching session with a research session if there was also a potential benefit to the learning experience of those taking part. For example, if students were given a questionnaire during a teaching session, preliminary information gained from that questionnaire about the whole cohort might allow tutors to better tailor a future teaching session to that group's needs.

Following presentations on Ethics at the Open University Doctorate in Education training weekend and consideration of the B.E.R.A. (2004) Revised Ethical Guidelines for Educational Research which have since been updated, the initial research proposal was completed and given approval by the Chair of the Social Science Research Committee prior to the administration of any questionnaires. A later proposal which provided more detail of the interview phase of the research was approved by the newly formed Foundation Centre Ethics Committee. A formal identification of the major ethical issues is given below using the categories given on my institution's ethics approval form, but it should be noted that this is not an exhaustive list of every tiny issue considered at every stage of the research which was implicit from my adoption of the Foundation ethos.

### Informed Consent

A full explanation of the purpose of the research, the purpose and structure of the questionnaire and the use which would be made of responses was explained both verbally and in writing. All students had the option whether or not to take part and to withdraw and completed a consent form.

A full explanation of the purpose and structure of interviews, including the use of audio recording was given both verbally and in writing. Students volunteered to

take part through completion of a consent form. Interviewees were selected from amongst these volunteers. After completion of the interview, participants were offered the opportunity to listen to and delete parts of the recording and later to read and/or amend the transcript. No one wished to do so. Students were also offered the opportunity to read through early versions of the thesis. All declined.

### Anonymity

All questionnaires were anonymous. The unique identifiers for each questionnaire (mothers' name and primary School) allowed matching between pairs of questionnaires (before and after teaching) but meant that only the respondents themselves were able to identify which papers they had completed. Where other information collated might enable identification, for example where the cohort for a particular degree route was small, questions were adjusted to merge cohorts. For example, Medicine, Biology and Biomedical Sciences were classed as one group so that the one student on medicine could not be identified.

Interviews were clearly not anonymous in that the interviewer knew who they were interviewing. Further, the interviewees identified their individual scripts as part of the interview process. At no point were the real names of the students recorded. Transcripts and matching questionnaire scripts were labelled by pseudonym.

### Confidentiality

The system of anonymity for questionnaires ensured that no privileged information existed. The original interview tapes are kept in a locked file and will be destroyed on acceptance of thesis to ensure that it is not possible to identify interviewees by voice. At no point were contributions of specific individuals discussed with others.

Some questionnaire respondents wished to talk through their responses with a

view to improving their learning but did not wish to take part in formal research interviews. This discussion did not form part of my research and no records were made of the discussion. Nor was information gained from these discussions referred to in this research.

### Do no harm

The issue of vulnerability of adult learners, particularly with regard to anxiety, has already been discussed elsewhere. This was heavily influential in decisions about the most appropriate methods for questionnaire implementation. In the administration of the first questionnaire, anxiety was additionally monitored through the use of an anxiety section.

A major impact of Ethics considerations was to ensure that interviews did not take place until the Summer by which time students had completed the Foundation Year and officially progressed to their new route. This ensured that there was no possibility of pressure or perceived pressure due to my role as tutor and assessor.

Whilst this chapter has focused mainly on the methodology and design of the research, the next chapter shifts the focus towards the implementation of the process.

## **Chapter Four – Methods and Implementation**

### **4.1 Questionnaire Phase**

#### ***Implementation***

The first questionnaire was administered to 93 students (78 home and 15 international) in their first mathematics session on the first teaching day of the course in October 2006. A follow-up questionnaire, with identical mathematics content but no attitudinal section and improved questions on qualifications, was offered to all students in the first teaching week in January 2007. The improved version of the questionnaire in two parts and with fewer mathematics questions was administered to 110 students (79 home and 31 international) in the first teaching week in October 2007 with a follow-up near the end of the course in April. The number of participants for the follow-up questionnaire was smaller than for the initial questionnaire, partly because of the timing and partly because it was only possible to use three of the four mathematics groups for practical reasons. By this stage, a number of students would also have left the course.

On completion of the first booklet in October 2007, students had been asked to write down the time before opening the second booklet. For the initial questionnaire, this indicated that all but two students had completed the first section with sufficient time to look back and check and therefore non-attempts at questions were most likely to indicate inability to answer rather than lack of time.

#### ***Appraisal of questionnaire process***

##### **Matching and Anonymity**

In the 2006 project, there had been occasional difficulty with matching initial and

follow-up questionnaires, either because primary schools had similar names or because students had given different responses at different times. Although there was a slight possibility that this might have been deliberate, it seemed more likely that some people had simply attended more than one primary school. In later questionnaires, I had introduced a second personal password (in line with bank accounts) and asked for 'mother's first name'. I avoided the term 'maiden name' because it might have been unfamiliar for international students. I was not too concerned about how the word 'first' was interpreted (forename or pre-marriage surname) provided each individual student interpreted it in the same way in both questionnaires. The use of the two identifiers was normally found to be sufficient to match scripts. When only one identifier appeared to match because of changes in schooling or changes from first to second names, the progression route and qualifications were used to confirm the matching. All pairs of questionnaire scripts were matched and students appeared confident that anonymity was preserved. Therefore, this could be claimed to be a successful and reliable method for script matching which would be appropriate for other research studies.

### Comparisons of Initial and Follow-up questionnaires

Preliminary analysis of the initial and follow-up questionnaires for the 2006 cohort showed that some students were electing to use different strategies for the follow-up questionnaires, perhaps perceiving the task as an exam not a questionnaire. It was possible that responses had been influenced by the recent January test week and, therefore, the follow-up questionnaire for the next cohort was moved to April. However, similar changes in behaviour were also noted for these students.

Question facilities were available for the whole cohort for both initial and follow-up questionnaires, but comparisons between them were not made because not all

students completed both activities. Additionally, I suspected that the very weak students were more likely to be amongst those who had left the course, adding a bias to results.

### Questionnaire confidence ratings

The confidence ratings were found to be a useful tool to interpret student behaviour. When the full range of options was used, these helped identify when different processes might be taking place, although they had little discriminatory value for the students who selected the same level for most questions. Although I have argued that some students seemed to equate the questionnaire with taking an exam, particularly in the follow-up event, for others, particularly in the initial questionnaire, the inclusion of these columns might have helped identify the process as research not exam and reduced anxiety.

## **4.2 Interview Phase**

### ***Preparation of students for feedback and interviews***

Immediately prior to the administration of the follow-up questionnaire, details of feedback methods and the request for research volunteers were explained both verbally and in writing, enabling people to complete the slip at the end of the questionnaire if they wished. Students were told about how a notional 'score' would be calculated using a 'score' of 1 for a correct answer and 0.5 for a partially correct answer. Those wishing to obtain their 'score' without releasing their identity were offered the opportunity to have their results listed by primary school identifier. People who wished to have an informal debrief were asked to quote their primary identifier so that the scripts would be available for discussion. Only



those volunteering to take part in a recorded research interview were asked to identify themselves, and therefore their scripts, by name.

In practice, two forms of interview took place: informal feedback debriefing sessions and pre-arranged structured research interviews.

### ***Informal Debriefing Sessions***

The initial and follow-up scripts of those who had indicated an interest in a debriefing session were put together. Pairs of scripts were studied briefly to identify areas of particular interest and rough notes made to act as prompts to maximise value should a student request an ad-hoc debriefing.

Informal debriefing sessions took place when interested students called in and asked to look again at their questionnaires. The purpose of these informal sessions was feedback for the student, but inevitably some of the areas of research interest were visited. Since these sessions did not have prior agreement for research use I did not directly use any information that was generated.

### ***Selection of Participants for Interview***

Earlier discussions on sampling had suggested that sometimes it was more appropriate to select a purposive rather than representative sample for interviewing. Students who managed every question perfectly on both questionnaires and showed no working appeared less likely to provide as many new insights as those who made errors, showed their working or changed their methods. During questionnaire analysis, those scripts containing non-standard responses for the snake/ratio/percentage/multiplication bigger questions were identified and cross-matched with the primary school identifiers of those who had volunteered to aid in decisions on interviewees. The number of people

volunteering for interview was far smaller than that predicted from previous research activities, possibly reflecting my reduced teaching role. The majority of those who did volunteer were from groups where I had been involved in some teaching and included future teachers and psychologists who might have been both interested in my research and have built up trust. Volunteers from other groups tended to be those who had answered every question correctly.

The ethics requirements from my department meant that I could only contact students after the exams had been completed, by which time non-local students had returned home reducing the potential interviewee pool further. Additionally, some students were now in holiday employment. Using the reasoning just given, I tried to contact those people whose scripts had a number of incorrect answers first. I interviewed the first five that agreed to come.

The use of a purposive rather than representative sample enabled me to identify patterns in behaviour that might not otherwise have been picked up. Further research might be required to identify how widespread such behaviour was for the rest of the cohort.

### ***The Interviews***

The research interviews followed the structure suggested in Section 3.4, preceded by a general introduction and briefing and confirmation that people were still happy for sessions to be recorded. People were asked to work through a series of problems giving verbal explanations (Appendix 1d). Following Siegler's (1988) method, the questions had been written on cards and were also read out. People were then shown their questionnaires and asked to talk through what they did and what they thought they were thinking at the time. Finally, people were asked

directly if there were any times when they were aware of either conflict or consolidation between work they were studying and ideas they already had.

### ***My Interview Role***

It was clear from the beginning that my role as both researcher and Lecturer would be a complex one. Neither I, nor the interviewees, would be able to divorce one role from another. Whilst I had some perceived power through my institutional role, and there was a risk of reactive effects (see Evans, 2000), the whole ethos of the Foundation Programme relied on mutual respect and partnership, working together to enable learning and often sharing high levels of humour along the way. By this stage in the year, the ethos would have been well-developed. The awareness that I too was on a learning journey, did not know all the answers and required their help to complete my work provided an element of levelling of power within the interview. The discussions were punctuated with bouts of laughter, sometimes anxious, and sharing of stories about life outside college. My trip to Specsavers the previous week appeared in two of the interview transcripts.

When interviewing children it might have been possible to ask questions, prompt for further clarification, note what occurred and then finish the interview. The children might not have an expectation of an outcome from the session. However, adults were expecting more of a two way process. They were willing to help me with my work where they knew more, but I was expected to help them in the mathematics work where I knew more. This was implicit in my reaction here:

*'I'm not going to rescue you just yet,,I want to see what you write in there.'*

In opening up to me the way that they were thinking, they wanted some affirmation of their thought processes. Was what they were doing acceptable? (Meeting

expectations, (Leron and Hazzan, 1997)) Sometimes this request was hidden in a self-denigration form that required a denial from me:

*'I might be wrong like!' or 'Sorry about my answers.'*

or *'I'm probably totally wrong because like I said I'm rubbish at maths.'*

Sometimes, the request for affirmation was more direct, in decreasing confidence:

*'That's right isn't it?' or 'Is that right?' or 'Am I wrong?'*

Sometimes, the request was more direct still:

*'What should the answer have been?'*

The transcripts are peppered with illustrations of positive re-enforcement, from the fairly standard:

*'Well done, yes that's right.'*

to the more effusive:

*'I loved your answers. I really did because you made it so much more interesting.'*

which was the response to the quote above about being rubbish at maths.

I also used the technique I find helpful in classes, which reduces the feelings of isolation or failure by pointing out that lots of other people have done the same:

*'Very few people got this right, the answer is actually multiply.'*

or *'A lot of people get confused with that one.'*

The excerpt below shows one of the most extreme examples of positive reinforcement against a negative self image of student. The subject under discussion is an algebra question.

Interviewer: *You got that that right.*

Elaine: *Really?*

Interviewer: *So you had latent algebra skills on that first day.*

Elaine: *I just thought, I haven't got any idea, because I'm thinking this is where I prove I'm thick and stupid.*

Interviewer: *And this one which is actually complex algebra you got it right.*

Elaine: *(laughter) I don't believe it.*

Interviewer: *That's brilliant !*

Elaine: *I haven't got a clue what I was doing, I must have been working it out as I was going along, but I haven't got a clue.*

Interviewer: *Yeah but you got it right.*

Elaine: *Same with that one, I just thought well it looks right.*

Interviewer: *You've got some latent algebra in there, that's impressive, are you impressed with yourself?*

Elaine: *I am, considering I got it right and I didn't have a clue what I was doing!*

On a few occasions I took a more direct teaching role, partly as 'payment' for them attending the interview. One student talked of a total lack of understanding about why she might have been taught the Normal Distribution, what it was and how she should use it. She voiced her concern that she would find real difficulty with this the following year and I responded by giving her a short lesson on it. Initially, I felt guilty about my 'over-involvement' in the interview but then found that Evans (2000) comments resonated with mine. He too talked of reverting to being a teacher at the end of interviews, of talking through the answers when requested by the student and of offering extra tutorials to help clear up misunderstandings.

There were also some occasions when I felt I adopted a more independent researcher role, using more neutral and standard interview phrases and feeding back to find more information.

## ***Appraisal of interview process***

Despite the difficulties highlighted above, or perhaps, in part, because of them, I generated five interview transcripts which were rich in data. Within them, there was a wealth of information about affective issues and a number of new ideas, some of which were unexpected.

I was not always convinced about the ability of students to look back and shed light on the processes they had used previously in their questionnaires, particularly when processes appeared to have large instinctive elements. Sometimes, the explanation provided by a student seemed very convoluted and suggesting an attempt to reconstruct (Duffin and Simpson, 2000; Karsenty, 2002), particularly because this explanation didn't seem to fit with the reasoning observed in other students who obtained identical incorrect answers. Sometimes students admitted they didn't know themselves and made guesses:

*'I think I put it down wrong or something, unless I thought it was 2.8'*

Sometimes they were rather more blunt!

*'Don't ask me what I did there'*

or *'Why on earth did I get that?'*

Although these responses might cast some doubt on the reliability of student memories and interpretations, in another way they were of major importance because they clearly highlighted that there were occasions when students acted instinctively but were unable to identify why they had done so (Fischbein, 1999). Reasons for actions were not explicit to the students themselves.

I could not assume that the methodologies used, no matter how ingenious, would be identical to those that might have been used were I not involved in observing the task. The need to explain to me verbally might have pushed people towards

oral rather than written methods and it has already been identified that people sometimes select a method they are able to explain (Kirk and Ashcraft, 2001). There might also have been a reluctance to report a method felt to be 'illicit' and elements of feeling the need to 'meet expectations' (Leron and Hazzan, 1997) in their choices. These issues needed to be born in mind when interpreting the data.

Each interview that took place was the product of interactions between myself and my student and developed from that partnership. I am aware that different group environments and alternative interview formats might have provided different insights, but I believe that the mutual trust between two adults and the shared learning journey that developed in these deep interviews enabled me to gain some unique and valuable insights.

## **4.3 Method of Transcript Analysis**

### ***Overview***

A balance needed to be created between the search for data on specific topics and ensuring that such a search was not so closed that potentially valuable insights which might be related to my research were lost. As a compromise, it was decided to use a somewhat iterative process. Initially, data would be extracted in response to specific areas generated from the research questions and associated theory. It was then intended to split these extracts further by allocating provisional coding suggested by the data and then regrouping into larger categories, a process with some similarities to grounded theory. The original transcripts would then be revisited to check that no additional excerpts or sub-sections had now become appropriate to include in the new categories. Originally, it was also intended to search for areas of repetition within the transcripts as a method to generate additional themes (see for example, Bryman, 2008) but this criterion

proved too difficult to implement in practice. Instead this was replaced by the notion of 'resonance', whether something appeared to resonate with theory already discussed or with ideas that had previously been generated.

Since lines of transcript had the potential to appear in more than one category and it was believed time consuming and a potential source of error to précis extracts to put into a table, the Framework method suggested in Section 3.4 was adapted and information stored in excel sheets, one for each category, rather than a smaller table with narrower columns. After the first transcript had been considered, it was identified that putting extracts in sections gave some useful insights but that the fragmented nature of some of these extracts occasionally prevented the identification of stories. Thus, a mix of extracts and story précis were used from this point. The analysis process used was believed to be appropriate for extracting most of the information relevant to the research area under consideration in this study. However, it was recognised that additional information relevant to other research areas might potentially be extracted in the future if the transcripts were revisited with a different set of first categories.

### ***From research questions to themes for extraction***

The questionnaires were expected to provide the major source of data for consideration of the first research question. It was hoped that the interview transcripts would provide additional information and some triangulation, so the choice of topics and later selection of sub themes was guided by the questionnaire findings and associated theory rather than emerging from the transcripts.

Excerpts and stories were collected together under the three mathematical areas:

- Percentage,
- Proportion



- 'Multiplication makes Bigger and Division makes Smaller'.

The second research question referred to mathematical behaviour and understanding. A major theme emerging from the theory was that of the interaction of mathematical behaviour with the affective domain. Essentially, answering the second research question required consideration of:

- What students were doing and
- Why they were doing it.

In the first sweep for these areas, all explanations of processes were extracted. These included both statements of explanation from the students such as, '*I guessed*' and extended stage by stage calculations. Some of the latter explanations were later précised, particularly if they overlapped considerably with the work already extracted for triangulation with the questionnaire. This section on processes produced far more extracts than for the other research question themes and it was in this section particularly that coding was used.

Once all the transcripts had been considered, extracts were provisionally coded using an additional column inserted on the spreadsheet. The source and formality of initial codes varied from short summaries of content to theoretical terms but the notion of resonance with theory or other data was used both for the initial coding and any later grouping. For example, one extract referred to '*what you wanted*' and clearly resonated with ideas of pleasing the authority figure. This was immediately labelled 'meeting expectations using the label from Leron and Hazzan (1997). Ann referred to getting lost and '*losing track*'. This was initially labelled 'confusion'. However, when another transcript also mentioned losing track this provided resonance and the coding for both was relabelled 'losing track'. When other excerpts initially labelled as 'misremembered number bond' and 'near miss'

resonated with theory and were grouped together as 'cognitive errors', 'losing track' joined that category too.

Some excerpts were very difficult to code initially and were simply left to be revisited as coding developed. Some labels were very tentative. Some excerpts were split or repeated to allow use of more than one code. Not all excerpts immediately resonated with theory and in some cases, rereading the transcript and context identified that initial coding was misleading. An important example of this occurred with the use of the code 'trial and error'. The finding that 'trial and error' seemed to be written against more excerpts than expected resulted in a revisiting of extracts previously labelled 'checking for reasonableness' and, hence, led to the emergence of the theory that trial and error was not simply a final method to use when all else failed but sometimes used as a first method. This was an example of ideas emerging from the data rather than being searched for from the theory.

The third research question focused on interaction of old and new knowledge. The last formal question in the interview had specifically asked interviewees if they were aware of any interactions, so it was expected that this section of interview scripts would be particularly valuable. However, it was also hoped that some interactions might arise more naturally throughout the interview.

- Sections of dialogue which showed a teaching intervention followed by a student reaction

were extracted for closer consideration. Additionally, excerpts which referred to

- school,
- teachers,
- work,
- memory

- ideas that had come from the past

were also extracted.

Provisional coding of extracts mentioning school or teachers soon identified two kinds of information. One was about what had been learnt in school and was, therefore, potentially relevant to this research question. The other was more about the school experience. This latter group of extracts were consequentially transferred to a new area of interest that had emerged as explained in the following paragraph.

One additional theme emerged based on 'resonance' with theory. As the transcripts were read, there were occasions when statements or stories from interviewees provided illustrations of attitudes or beliefs. Though not unexpected, these provided additional evidence for the earlier assertions made about Foundation learners which were used to inform the development of the research questions and research design.

It should be noted that interaction with literature was not necessarily always a linear process. There were occasions when ideas emerging from the transcripts prompted a revisiting of literature sources and a consequential expansion or deepening of the literature review. Identification of 'losing track' for example resulted in more targeted reading on working memory, followed by a greater emphasis on this topic within Chapter 2. Similarly, insights from the transcripts sometimes prompted a revisiting of questionnaire scripts to identify if additional evidence could be found from the wider cohort to support emergent theories. For example, the identification that students were using number to select solution method prompted a closer look at the differences in methods used for two mathematically similar problems.

## 4.4 Summary

Questionnaires were administered to 157 adult home students and 46 international students on the first teaching day of their Foundation Course in October 2006 or 2007. Follow-up questionnaires, with identical mathematics questions were then administered to the same cohorts later in their course. Initial questionnaires were also administered to some students in October 2008. The main mathematical topics considered were Percentage Use, Proportional Reasoning and the over-generalisation, 'Multiplication makes Bigger, Division makes Smaller'. Students were asked to indicate their confidence in answers. Initial and follow-up questionnaires were linked by personal identifiers, primary school and, for the second cohort, additionally mother's first name. This proved a successful matching method. International student responses are not included in this study.

Five students, selected from a pool of volunteers using purposive sampling, took part in semi-structured interviews. Interviews were recorded, transcribed and then analysed using the method described in Section 4.3.

The process of analysis of questionnaire scripts and interview transcripts and interaction with literature was not a linear one. Valuable insights emerging from one source informed the search for insights within another. The results of this analysis are presented in the next three chapters. Chapters Five, Six and Seven each focus on different aspects of the data related to one of the three research questions.

## **Chapter Five – Comparisons of Understanding**

### **5.1 Introduction**

The claim was made on a number of occasions within Chapter 2, that the mathematics knowledge base of adults might be different from that of children. Hence, this was the focus of the first research question. This chapter sets out to explore some of those differences by comparing the responses of Foundation Programme adults to those of children in the CSMS survey (Hart, 1981a) using similar questions. Whilst most of the discussion that follows emerged from questionnaire analysis for the 2006 and 2007 Foundation cohort, there were occasions when events within the interviews provided additional support or further insights into activity. Thus, some interview extracts have also been included. Additionally, some results from the 2008 cohort were used in Section 5.4. In the 2006 study, additional questions were asked on Directed Number, Place Value and Algebra (Dodd, 2007a) but these were not included here.

As a first stage, the facilities for all the questions given to the Foundation cohorts were directly compared with facilities for similar CSMS questions placed in order. Although the resulting hierarchies of the whole cohort could be argued to be very broadly similar, closer looks at individual students showed much more variation. If students who answered everything, or nearly everything, correctly were removed from the mix, those left seemed to have a more mixed set of achievements, linking more with Coben's (2000) confused mental map. Extremes included the student in 2006 who successfully answered most of the algebra questions but was unable to find 10% of 7.75. In the following analysis it was often the differences between the CSMS and the Foundation cohort hierarchies that indicated areas worthy of further investigation, rather than comparisons of actual facilities themselves.

## 5.2 Ratio and Proportion

Four sets of proportion type questions were given. One set involved the allocation of mice to snakes in proportion, another used recipe proportions and the third required the cost of a number of items. The fourth set which involved proportions in a sample was only used in 2006. Detail of responses to ratio questions is given in Appendix 3.

### **Snakes**

Four questions were asked on snakes (Figure 5.1). These questions were very similar to those used in the CSMS survey (Hart, 1981b).

Figure 5.1: *Questionnaire Questions on Snakes*

Question 14

Three snakes are kept in captivity and fed on mice.

The number of mice they are given depends on their length.

A \_\_\_\_\_ 50cm

B \_\_\_\_\_ 100cm

C \_\_\_\_\_ 150cm

a) If snake A is fed 2 mice.

(i) How many would snake B get?

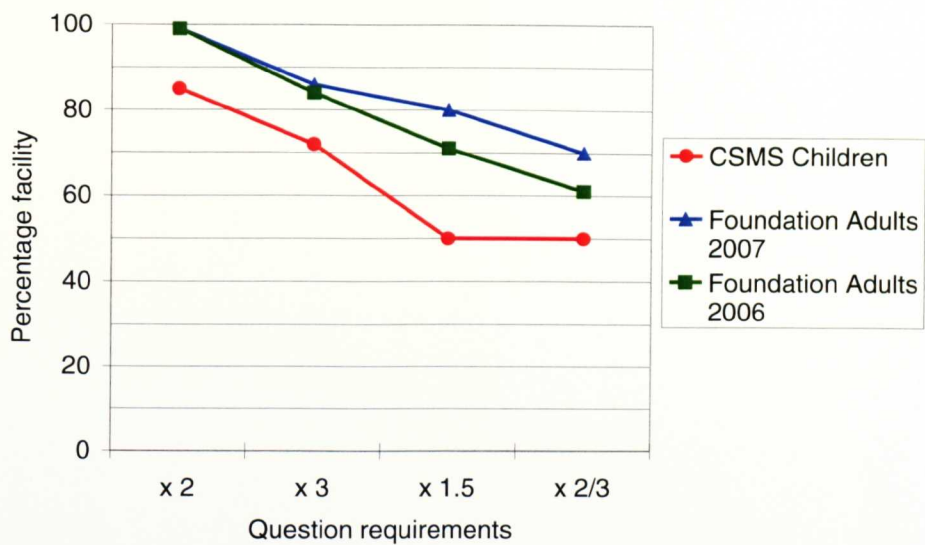
(ii) How many would snake C get?

b) If snake B is fed 6 mice, how many should snake C get?

c) If snake C is fed 12 mice, how many should snake B get?

All students attempted the questions and notes on papers or later interviews suggested that as with the CSMS children (Hart, 1981b), a number were using the informal strategies (Lamon, 1994) of halving, doubling and building. The results are shown in Figure 5.2. The numbers used for the CSMS questions and the foundation students were not the same, but if it is assumed that this did not make any difference, Foundation adults performed slightly better than CSMS children. Both groups maintained the same question hierarchy with decreasing success as the multiplier became more complex.

Figure 5.2: Graph to show percentage facilities for questions on snakes for adult foundation students and eels for CSMS children ( Hart, 1981b, p.98)



It was noted that the drop in facility for adults between part b and part c was not mirrored by the answers from the whole CSMS cohort, although facilities for younger children did show some drop. However, it was later noted that **8%** of adults who correctly answered the question requiring a multiplication of 1.5 gave the incorrect answer of 9 for  $\frac{2}{3}$  of 12. This interchanging of 8 and 9 for  $\frac{2}{3}$  and  $\frac{3}{4}$  of 12 appeared in a number of questions and the phenomenon is studied in more detail in Section 6.5.

For the question a) parts (i) and (ii), it was possible to obtain the correct answer by using a correct proportion method (doubling or tripling) or the strategy 'add on two'. Apart from the occasional x 2 and x 3 written on scripts it couldn't be confirmed which methods people were using. However, all five interviewees used terms like doubling and times 3 and had no problems with the questions. The slight drop in facility between the x 2 and x 3 corresponded to **7** students in 2007 and **10** students in 2006 who gave the answer '5'. It was possible that they had selected the answer 5 as larger than 4 or as some sort of '1 more increase' as suggested by Hart (1981b). However, 5 is also the result of  $2 + 3$  raising the possibility of a retrieval error. One



student pointed out that if you keep snakes, you don't actually feed them in proportion and perhaps the answer 5 reflected this knowledge (see earlier discussion on pseudo real life questions (Maier, 1991)).

Part b required a more complex two stage process. This could be an identification of the multiplier 1.5 (probably instinctively) followed by the multiplication or the recognition that a 50 cm snake would require 3 mice so a 150 cm snake would require 9 mice ( either  $3 \times 3$  'find the rate' (Hart, 1981b) or  $6 + 3$  building up). **19%** of adults in 2006 and **13%** of adults in 2007 gave the incorrect answer of 8, possibly using an 'add on two rule' to arrive at this answer. An example is shown in Figure 5.3. Hart (1981b) had identified similar issues.

Figure 5.3: *Example of student use of 'add on two' rule*

b) If snake B is fed 6 mice, how many should snake C get?

6 mice = 100cm.  
 $150 - 100 = 50\text{cm.}$   
 $100 - 50 = 2.$   
 $6 + 2 = 8$       8 mice.

Brenda used a rate method for the interview eel question and explained:

*'oh right, so if B eats 12 then what I'd have to do is go back to C. If B is having twice as much as C, then C is 6 so 6 times 3 is 18'.*

Christine elected to build:

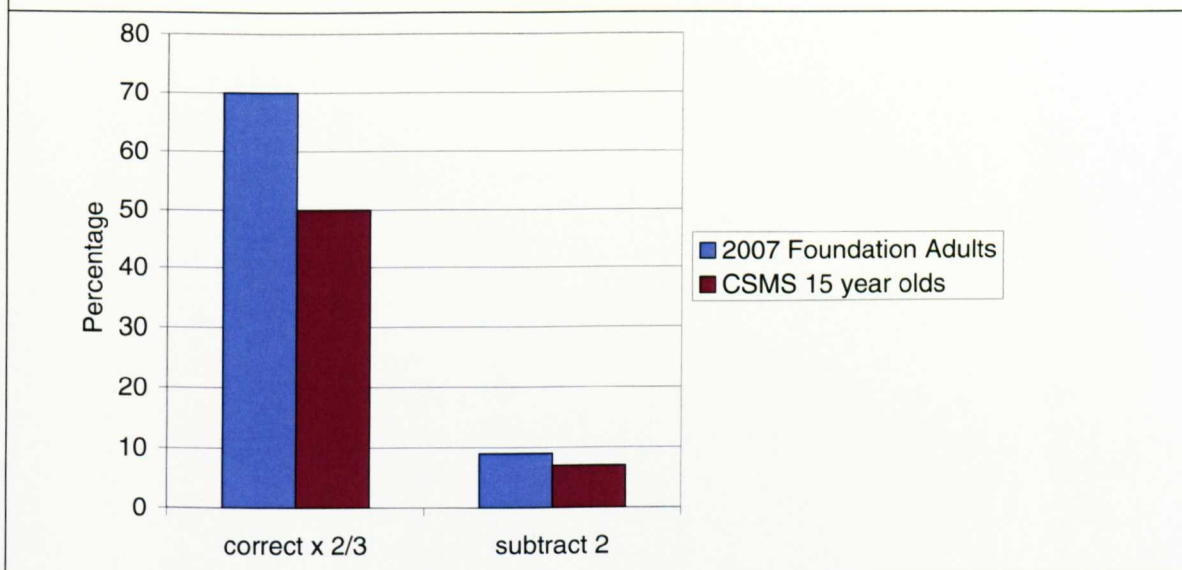
*'B is half as big as C, so I need half as much of 12, so from the 12, I need another 6 added on to it to give me the 18.'*

Part c required a multiplication of  $\frac{2}{3}$ . The strategy of halving and adding used successfully by some adults and children for the previous question could not be used for this one. **11** of the adults in the two cohorts gave the answer 10 corresponding to Hart's (1981b) suggestion of a subtraction of 2. The fact that **6** out of the **7** students in 2007 who incorrectly gave 10 also incorrectly gave 8 ( $+ 2$



strategy) for the previous question supported this theory. Comparison of results with the CSMS cohort (Figure 5.4) indicated that, although a higher proportion of adults gave the correct answer for a snake  $\frac{2}{3}$  the size, a higher proportion of adults also gave the incorrect answer of 10 (or CSMS equivalent).

Figure 5.4: *Percentages of Adults and CSMS Children (Hart, 1981b, p.92) answering snake/eel question correctly by ratio and incorrectly by subtraction.*



Of those interviewed, only Elaine gave the answer 10 in both questionnaires. Her explanation did not imply that she was simply subtracting 2, although, given her confusion trying to make sense retrospectively, that may well have been what she had actually done:

Elaine: *Well because the first snake C is 150 cm and it got 12 mice and B would get half plus a quarter, would get three quarters if you like.*

Interviewer: *Right.*

Elaine: *So it would get 10.*

Interviewer: *Can you run me through again, how did you get the 10? If you can work out how you got the 10.*

Elaine: *I don't know if that makes sense but it's the way I worked it out.*

Interviewer: *Half is?*

Elaine: *Half is 6.*

Interviewer: *Half is 6 and a quarter is...*

Elaine: *I was sort of thinking..... I should have put in 9 not 10.*

I wasn't convinced that this reflected her real actions:

Interviewer: *9 is actually the correct answer, did you think that's actually what you did to get the 10?*

Elaine: *No that's how I worked it. I thought because It's not double its size and it's not half its size so I took half the 6 and then half the 6 again and I should have put nine and I think using the quarter that I added 4 to the 6 instead of 3.*

I thought that it might have been helpful to look at the method Elaine had used for the eels in the interview to see if this shed some light on her real methodology:

Elaine: *B's got 2 , C should get 1 and A should get 3.*

Interviewer: *Brilliant ok next bit.*

Elaine: *If B gets 12, how many should A be fed? 12 .....18*

Interviewer: *Well done.*

Elaine: *If A gets 9, how many should B get? 6 and 3 quarters (laughs)*

Interviewer: *Run me through that again, what did you do?*

Elaine: *Well it got 9, if A's got 9, so B should get half plus a bit more because there is only 5 between them, so he should get sort of if he was half he would be 20 and he would be 10, that's the way I'm thinking so, if he gets 4 and a half plus half of 4 and a half, which is 2 and a quarter so that makes 6 and three quarters. (laughter)*

Interviewer: *I just imagine me having some fun sorting the transcript of this.*

Elaine:        *You'll be saying, 'I know who did that'.*

Interviewer: *So basically the eels won't go hungry.*

***Recipe Questions***

Two sets of questions about recipes were given in the questionnaire.

Figure 5.5: *Questionnaire Question 20 on Recipe*

Question 20.    A caterer has a recipe for 80 people.

                  It requires: 12 eggs,    2 lemons,     $\frac{1}{2}$  box of packet mix

                  For 40 people:    How many eggs would you need?

  How many lemons would you need?

  How much packet mix would you need?

                  For 60 people:    How many eggs would you need?

  How many lemons would you need?

  How much packet mix would you need?

and later

Figure 5.6: *Questionnaire Question 23 on Recipe*

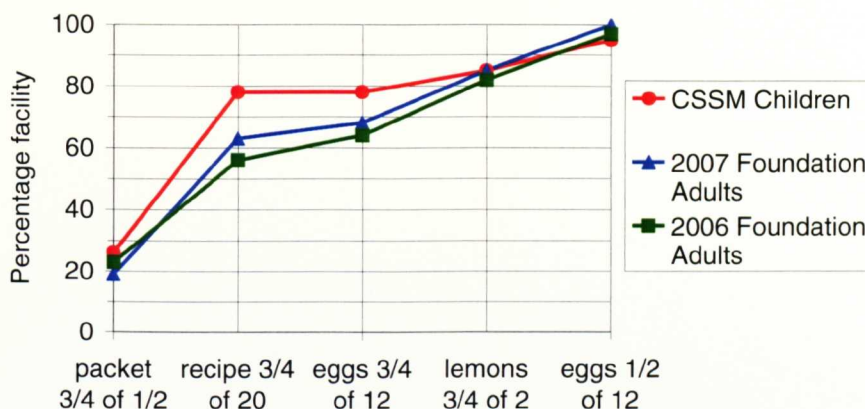
Question 23.

A caterer has another recipe which requires him to use 20 spoonfuls for 8 people.

How much for 6 people?                    How much for 15 people?

The CSMS survey gave an amount for 8 people and asked for the amount for 4 and 6 people (Hart, 1981b, p.90). This included an original amount of 2 and a  $\frac{1}{2}$  pt of cream. A comparison of these results for adults and children is shown in Figure 5.7. Hart suggested that, where possible, children used strategies of halving and doubling, building answers by putting parts together, or as Lamon (1994) says using composite units, and avoiding using fractions when they could. The similarity between facilities for the different cohorts finding  $\frac{1}{2}$  of a whole number and  $\frac{3}{4}$  of 2

Figure 5.7: Graph to show percentage facilities for recipe ratio questions for adult foundation students and CSMS children (Hart, 1981b, p.98).  
(note: integer numbers used in CSMS questions were different)



suggests that adults were using similar methods. Occasional working on questionnaires reinforced this view.

It was noticed that **20%** adults in 2007 and **26%** adults in 2006 gave an answer of 8 instead of 9 for  $\frac{3}{4}$  of 12, indicating again some form of instinctive error rather than difficulties with calculation (see Section 6.5).

If this explained the difference in facility between adult and child cohorts for this question, it would imply that some adults were doing a fractional calculation, not finding  $\frac{1}{2}$  and  $\frac{1}{4}$  and adding. This instinctive error would not explain the drop in facility for the later question requiring  $\frac{3}{4}$  of 20, but the difference here might be caused by semantic influences (Kaput and West, 1994). Students were not asked to find the amount for 4 first and this lack of build up in questions would remove scaffolding.

For the question requiring  $\frac{3}{4}$  of  $\frac{1}{2}$  the incorrect answer of  $\frac{1}{3}$  was given by **27%** of adults in 2007, **29%** of the adults in 2006 and **20%** of children. Hart suggested finding  $\frac{1}{2}$  of  $\frac{1}{2}$  and then adding  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{2}$  was 'cumbersome' (Hart, 1981b, p.90) and that people

needed to change method. She also stated that no-one in the CSMS interviews multiplied by a fraction. Ann certainly attempted to multiply by a fraction:

*'For 60 people, that would be  $\frac{3}{4}$  so I would...  $\frac{3}{4}$  of 12 would be (laughs) yeah, so  $\frac{3}{4}$  (mumbles) would give me 9 eggs. Lemons,.. that's a bit more tricky! Oh no. If I divide that by 4, I get  $\frac{1}{2}$ , so 3 times  $\frac{1}{2}$  is one and a half lemons.'*

although she recognised that she had gone wrong somewhere:

*'Here we go, half a box of packet mix and I need  $\frac{3}{4}$  of a half ...(long pause) so if I divided a half by 4, I would get eighths and I'd want  $\frac{3}{4}$  of 8 oh I'd want six eighths, so I'd want  $\frac{6}{8}$  of a packet. No I wouldn't cause that's more than a half. I don't know!' (laughter)*

Hart (1981b) suggested that children gave an answer of  $\frac{1}{3}$  because they were seeking a number between  $\frac{1}{4}$  and  $\frac{1}{2}$ . This extract from the interview with Diane supports the notion of guesswork for some adults too:

Interviewer: *Explain to me how you got that one, one third ?*

Diane: *80 people, 60 people, I don't know? I'm guessing now it should have been a quarter.*

Interviewer: *Right, so you can't think how you got that one?*

Diane: *Well this is a recipe for 80 people so for 60 people.. no I don't know.*

Interviewer: *Right, 40 people. You have got a quarter...*

Diane: *(Questioning) It should be more right?*

Interviewer: *(Confirming) It should be more than a quarter.*

Diane: *Should be more than a quarter...should it be two thirds then?*

There appeared to be evidence of some adults of situating themselves in real life, resulting in a level of estimating (Maier, 1991; Roth, 1999). One student quoted, “plus one for the pot” on their questionnaire and another pointed out that they would not calculate  $\frac{3}{4}$  of  $\frac{1}{2}$  in real life but calculate the number of grams. In her interview, Elaine said:

*‘I was thinking ‘How do I split half a box of packet mix into 3?’*

Coping with fractional quantities in real life recipes was not realistic and if near misses, i.e. sensible quantities, were accepted, adults did far better than children.

Question 23 which asked to find the number of spoonfuls for 6 and then 15 people if 8 people needed 20 spoonfuls, gave some additional insights. The amount for 6 people could be found using the strategy of halving and adding as in the other recipe question, but correct answers were fewer, again possibly reflecting lack of scaffolding. Some people appeared to add on one extra person rather than an extra spoonful.

Elaine gave the correct answer of 15 for 6 people in her first questionnaire and the incorrect answer 18 in her follow-up questionnaire, identifying both answers as just guesses. She was asked to decide which one was right:

*‘So for 4 people it would be 10....so for 6 it would be 15.’*

When she looks at her incorrect answer she suggested:

*‘I think what I’ve done is just added the 8, I’ve gone it’s 8 people it’s 18’*

Slightly concerning though is the fact that the incorrect answer came on the follow-up not the initial. However, given both are guesses, could this simply be a ‘losing track’ problem (see Section 6.5 for discussion on cognitive errors)?

Finding the number of spoonfuls for 15 people required a much more complex methodology. The question might have leant itself to the unitary method, ‘how

much for one' or other variations of rate, but this required a calculation of 20 divided by 8 which was beyond what most people could hold in their head. As a result there were some interesting answers.

Diane attempted to find the rate for one person with more success in one questionnaire than another. In her interview discussion she tried to make sense of what she had done, and identified a method she felt might have been better; finding the rate for two (i.e. a composite unit (Lamon, 1994)) and then building up:

*'to be honest it would have been easier just to say, well I don't know, I had a method of doing it then. I was like it must be 15 because like... wait, if you divide this so it's like 5 spoonfuls for every 2 people that's how I would work it out just initially, but thought 'why have you got this?' well it must be 15 because that's like I don't know whatever of that and 6 is also a what's it called like a denominator of that or something so I mean it can be times.'*

The rest of Diane's explanation was somewhat confusing but it was not clear whether this represented her confusion when attempting the questionnaire or her confusion in trying to identify what she had done.

Using Hart's (1981b) explanation of the confusion between spoonfuls and people again, a number of incorrect answers such as 39 made from  $2 \times 20 - 1$  people or 27 made from  $20 + 7$  people might be expected.

Elaine gave the answer 39 in her initial survey and indeed stated:

*'I doubled it and took off 1.'*

She realised the answer was not correct, although her next suggestion was not a great improvement!

*'I should have doubled it and taken 2 off.'*

Her further explanation confirmed the confusion between spoonfuls and people as

she interchanged them:

*'I've doubled to 16 (presumably she means people) and then took 1 off (people or spoons) when it should have been 38 (spoons) is it?'*

Following some discussion where I stepped into a teaching not researcher role, she managed to make sense of this herself:

*'Yeah that's basically what I've done, I've doubled it and thought well 16 is 40 so 15 must and I've taken a person off. When I should have worked out how much sugar 1 person should get I've taken 1 person off.'*

Had she really understood? It appeared so.

Interviewer: *Back with the snakes, I wondered if that was what you had done there.*

Elaine: *I realise that should have been 9 now.*

Elaine's responses earlier in the interview showed that there had been other occasions where she had attempted to add or subtract and got confused about what she was adding. She identified that 40 people would need 6 eggs and wanted the number for 60. She said:

*'I keep thinking a quarter and adding 4 to the 6 to make 10.'*

It was not clear whether the quarter came from the packet mix answer just above, but whether or not her explanation represented what she was really doing she had managed to get the same answer of 10 on both papers.

### ***Cost of a number of items***

It was once a popular school task to set questions which required the cost of a number of items, when the cost of a different number of items was known. Kaput and West (1994) argue that semantically, price based questions are often easier to solve than other proportion based questions, because their situating provides



scaffolding or pushes solvers towards ideas of rate, 'How much for one?'.

However, calculating costs has some similarities with percentage use (Dowling, 1991) in that this may no longer be an everyday task. Many supermarkets now give the price per kg for a product for comparison which, whilst highlighting the notion of rate or in effect the unitary method, makes the requirement to do such calculations redundant and weakens the connection between the rate and its derivation from original information. It has also already been noted from Lave's (1988) work that people often make qualitative decisions about best buys etc.

The first section contained one question on the cost of items (see Figure 5.8).

Figure 5.8: *Question 18 on cost of Items.*

Question 18      8 cards cost 48p. How much would 12 cost?

**76%** of those in 2007 gave the correct answer of 72p for the first question which seemed surprisingly low but some people wrote the calculation  $12 \times 6$  and gave an incorrect answer. Elaine was one of these. She gave the answer 72 in the initial questionnaire but  $12 \times 6 = 60$  in the follow up. Her explanation:

*'Yes and don't ask me how I got 60, I probably was panicking. I do with papers and maths, because it's 72 not 60 isn't it.'*

Hidden in this response is some indication that she felt more pressured in the follow-up than initial questionnaire, indicating yet again that people equated the follow-up with a test.

Three people calculated  $12 \times 8$  and 2 people  $12 \times 48$  in 2007 which was perhaps more surprising. However, the overall facility for this question was higher than the 47% who worked out the recipe question needing the unitary method, possibly reflecting more familiarity with the need to find the cost of one. The question:

*‘If you buy five xmas cards for 65p, how much is each card costing you?’*

was asked in the ACACE Gallop survey (Sewell, 1981) and by Evans (2000).

Evans also asked the direct question  $91 \div 7$ . The results are shown in Table 5.1.

Table 5.1: <i>Table comparing success for cost type questions</i>			
Question	$91 \div 7$	Cost of 1 if 5 cost 65p	Cost of 8 if 6 cost 48p
Evans (2000)	90%	88%	
ACACE (Sewell, 1981)		77 %	
Foundation (2007)			76 %

The foundation results seemed similar to the ACACE result for the less complex question. If the questions were ordered by complexity,  $91 \div 7$  was the least complex because the maths was already abstracted and the instruction  $91 \div 7$  was given directly.  $65 \div 5$  was more complex since the calculation to be done had to be abstracted before calculation. The cost of 8 cards was the most complex since a decision had to be made to calculate the cost of one by dividing and then the cost of 8 by multiplying (or some alternative method). If the questions were ordered by arithmetic difficulty,  $65 \div 5$  and  $91 \div 7$  were more difficult than  $48 \div 6$  since the latter is in the common core of tables learnt and the two former questions require some form of mental long division or similar. The low facility for this question despite the lower arithmetic demand suggested that this question was worthy of further study and is considered in detail in Section 6.2.

The 2006 questionnaires also contained questions on sample constituents which had much lower facilities. Foundation adults performed slightly better than CSMS

children for the simpler question (**36% versus 32%**) and very much better for the more complex question (**29% versus 13%**) but it was not clear why the responses for the two cohorts were so different (Hart, 1981b, p.98). I was unaware of ways that helpful skills might have been consolidated in adult life and suspected that differences might be due to improved skills in comprehension or logical thinking.

### **5.3 Use of percentage calculations**

In the CSMS book (Hart, 1981b), percentage work was included in the chapter on ratio and proportion. These topics might appear to require similar strategies in school mathematics, but percentage calculations for adults frequently take place in shops whilst many ratio calculations take place in other everyday environments. I selected to consider percentage separately from ratio since these differences in adult contact might be relevant in interpreting differences in adult behaviour.

Dowling (1991) suggested that not all forms of percentage calculation are really used in everyday life. Sales shopping and simple estimating may be common, but calculating complex percentages and interest are often left to the lender rather than borrower with the mathematics itself computed via some software package. With the naïve strategies of relative frequency, it is possible to identify that one interest rate appears better than another (Sewell, 1981) and this is sufficient for some people to make their decision. The calculation of VAT which used to be a somewhat tedious process has been replaced by financial software for many. For those without the software, various street algorithms exist for calculating 17.5% which do not directly relate to standard percentage calculations. On a number of occasions, I have been asked to explain to adults why their shortcut works.

## ***Common Percentages***

In the analysis of the initial questionnaire answers, the link between common percentages and fractions was not always shown. Only **78%** of 2006 adults stated directly that 25% was a quarter, although it is possible that automotive processes were rejected and the result  $\frac{25}{100}$  given instead because of the perceived need to justify the answer (Kirk & Ashcraft, 2001). For 2007 most students were able to correctly write 25% as fractions, in some form, although **3** students incorrectly identified  $\frac{1}{25}$  as 25%. Some older students confidently answered this, but knew little else on percentages. Some students knew everything except this.

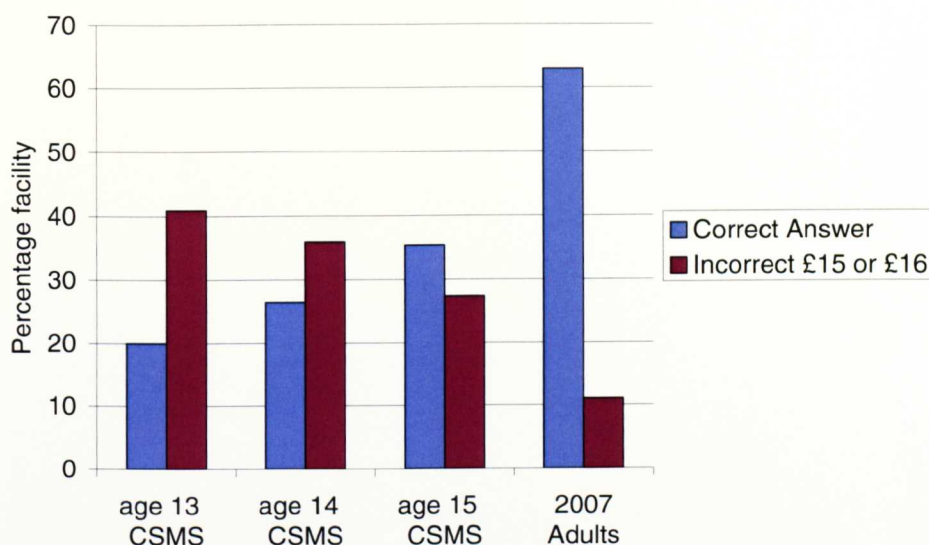
At first glance, comparison with the results from ACAME Gallop survey (Sewell, 1981) suggest that this is higher than for the general adult population. However, closer study and recognition of the reduced success rate for Evans' (2000) students, suggests that the lower results were a function of the 'trick' nature of the ACAME question, asking the percentage left, and a set of multiple choice options which did not include  $\frac{1}{25}$ .

In 2006 only **58%** of students correctly found 10% of 7.75. Some seemed unaware that 10% was  $\frac{1}{10}$  and carried out complex calculations first. Others had difficulties with place value. The 2007 cohort, had a much greater success rate (nearly all) finding 10% on its own, when the question was split into two parts.

## ***Percentage Problems - A% of B***

Of the percentage calculation questions, the question with the highest facility for adults was a problem asking to find the cost of a £20 picture after a 5% reduction. The results are shown in Figure 5.9. The graph shows that the most common

Figure 5.9: *Graph to show percentage facilities for different cohorts answering £20 reduced by 5% (Hart, 1981b, p. 96)*



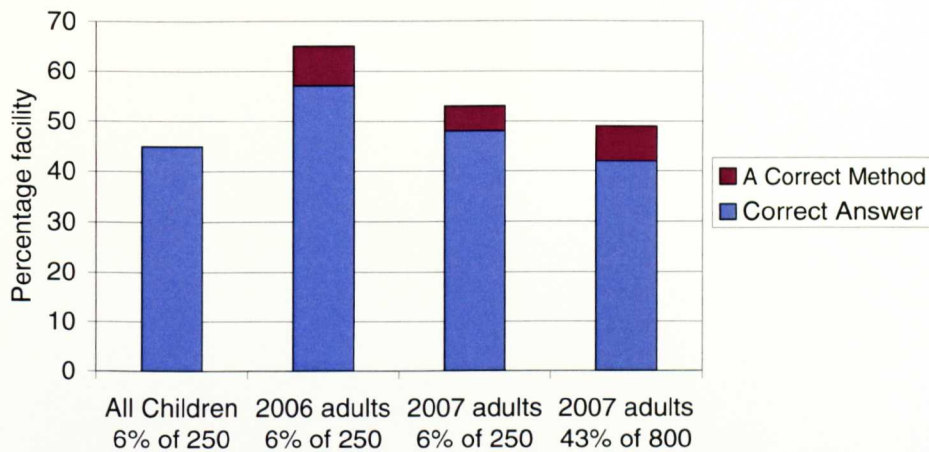
incorrect answers for children were £15 and £16 which Hart (1981b) suggested were found by children ignoring both the £ and % signs and just following the word 'reduced'. A few adults also did this but far fewer than the children. Christine confirmed that this is what she had done in her first questionnaire:

*'Yes, I've divided that 5 by there to give me 4 and then I have subtracted it, without even cutting it or thinking about the percentage, and yet I just did it in my head then.'*

The most popular method used by adults was to find 10% and then half it for 5%. If the students who gave the answer £1 or a correct method were also included the facility was even higher suggesting that 'real life' may have increased adult skills.

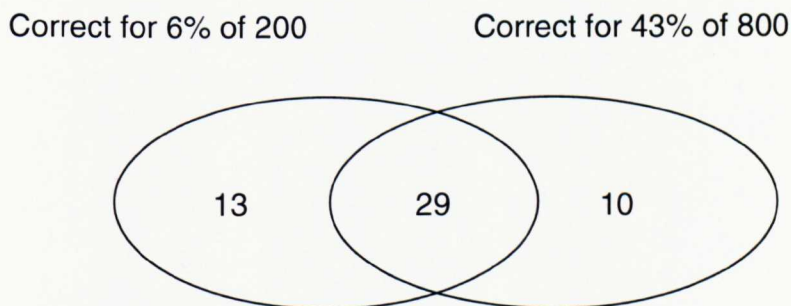
The results for other questions involving finding A% of B are shown in Figure 5.10. In 2006, the most successful adults used a similar strategy to that used for the picture problem (finding 10%, then 5%, then 1% and building up to find 6%). This strategy was also common for a question requiring the finding of 15%.

Figure 5.10: *Graph to show percentage facilities for ‘find A% of B’ for adult foundation students and CSMS children (Hart, 1981b, p. 98)*



In 2007, the 15% question was replaced by one requiring 43% and this seemed to cause more confusion in method selection. Adults who successfully found 6% of 250 could not always find 43% of 800 and vice versa as Figure 5.11 shows but, as with other kinds of ratio questions, this might also have been influenced by the different order in which numbers appeared (Kaput and West, 1994).

Figure 5.11: *Diagram to show numbers of people in groups answering questions correctly (2007)*



It was noted that a number of students changed their answers or methodologies between initial and follow-up questionnaires and some had moved from the use of

the 'build up methods' to the one stage methods more appropriate for calculator use, not always with equal success.

Ann clearly identified that her mathematics was situated within the context (Lave, 1988; Maier, 1991; Roth, 1999). She would do one calculation in a shop and a different one in an exam.

Interviewer: *That one again, how would you do 43% of 800?*

Ann: *I would - it depends what I was doing it for, if I was doing it for an exam like this, I would do my  $43/800$  times 100 but if I was just sort of out and about I would take it to 50%, of that, so I would half that and know it's slightly less than that for a rougher figure.*

This was a clear example of different acceptable outcomes in different situations (Maier, 1991). Evans (2000) noticed a change in positioning, from everyday maths to school maths, for the majority of his interviewees when he phrased a 9% increase as, 'About how much will that be?' then 'Can you work it out exactly?' (Evans, 2000, p.254). For his students the trigger appeared to be 'about' to 'exact'. For Ann, the link was in the other direction, the placing of the question identified whether it should be exact or about.

### ***Other Types of Percentage Problems***

For other types of percentage problems, adults in the initial questionnaire did not perform significantly better than children. For the CSSM children, results for the problem 'A% of B' and 'find A out of B as a percentage' were similar, **(39%)**. For the adults in 2006, results dropped from **74%** for the former to **43%** for the latter, perhaps reflecting my suggestion that one type of problem was met more frequently than another in real life. The drop in confidence ratings by individual students between questions indicated that students recognised problems were of



a different type even if they couldn't do them or had forgotten methods.

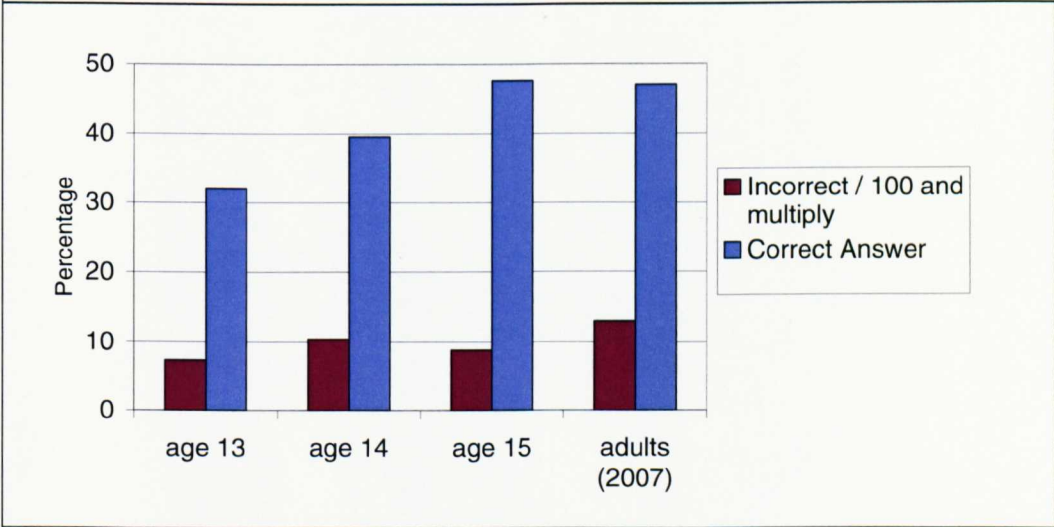
Some students attempted to use the same method to solve both types of problems. One explanation could be some people believing that percentage calculations always involved dividing by 100 as Hart (1981b) had observed amongst children. Elaine certainly believed this:

*'I don't remember what you do with them, do you multiply them by 100?'*

(see Elaine's story, Section 7.3). However, the confidence drops for most students suggest their attempts were not strictly over-generalising (see Section 2.2) because they recognised that there were limits to their domain (Swan, 2001).

The results for the question asking to give a proportion as a percentage are shown in Figure 5.12. The proportion of adults in 2007 incorrectly selecting to divide by

Figure 5.12: *Graph to show percentages of cohorts who correctly gave A out of B as a percentage. (Hart, 1981b, p. 96)*



100 and multiply was larger than the proportion of children who used this strategy. Some adults used the same method whatever the problem gaining success when this method was appropriate and failure when it was not. Some adults, including Diane and Christine appeared to use a kind of build and match method though not



always successfully. Diane's answer to question 25 from the first questionnaire is shown in Figure 5.13.

Figure 5.13: Example of Diane's method of 'Build and Match'

**24 birds in a colony of 300 have inherited a particular characteristic. What percentage is this?**

$10\% = 30$   
 $1\% = 3$   
 $2\% = 6$   
 $30 - 6 = 24$   
 $10 - 2 = 8\%$   
8%

She had calculated that:

10% was 30 and 2% was 6,

therefore,

24 which was  $(30 - 6)$  must be equivalent to  $(10\% - 2\%)$ , i.e. 8%

Christine, less successfully identified:

10% was 30, 5% was 15 and 1% (although this was really 2.5%) was 7.5

Therefore,

24 which was  $(15 + 9)$  must be  $(5 + 1 + \dots)$  i.e. 'less than 7%'

Figure 5.14 shows another extract of interest from a questionnaire. The student

Figure 5.14: Example of continuing for reasonableness

**In a group of 600 mice, 42 have blue eyes. What percentage is this?**

$600 \div 42$   
 $42 \div 1.2 = 35$   
 $35 \div 4 = 8.75$   
 $8.75 \div 100 = 0.875$   
100  
 approx 10%

had attempted to find a solution, but when they were unable to calculate one had resorted to providing an estimate rather than simply abandoning the question, possibly a demonstration of adults continuing until they had found a solution based on 'reasonableness' (Benn, 1997).

The CSMS survey (Hart, 1981b) suggested that some children might use  $a/b = c/d$  to calculate percentage but that this usage seemed rare. No home students used this method. However, nearly every international or overseas student in 2006 used this method, generally successfully, and for every percentage calculation.

### 5.4 Does Multiplication Make Things Bigger?

Three questions in the questionnaire asked which calculation gave the bigger answer (Figure 5.15). For the 2007 cohort, **99%** of students correctly selected the

Figure 5.15: *Questionnaire questions on which calculation gives a bigger answer.*

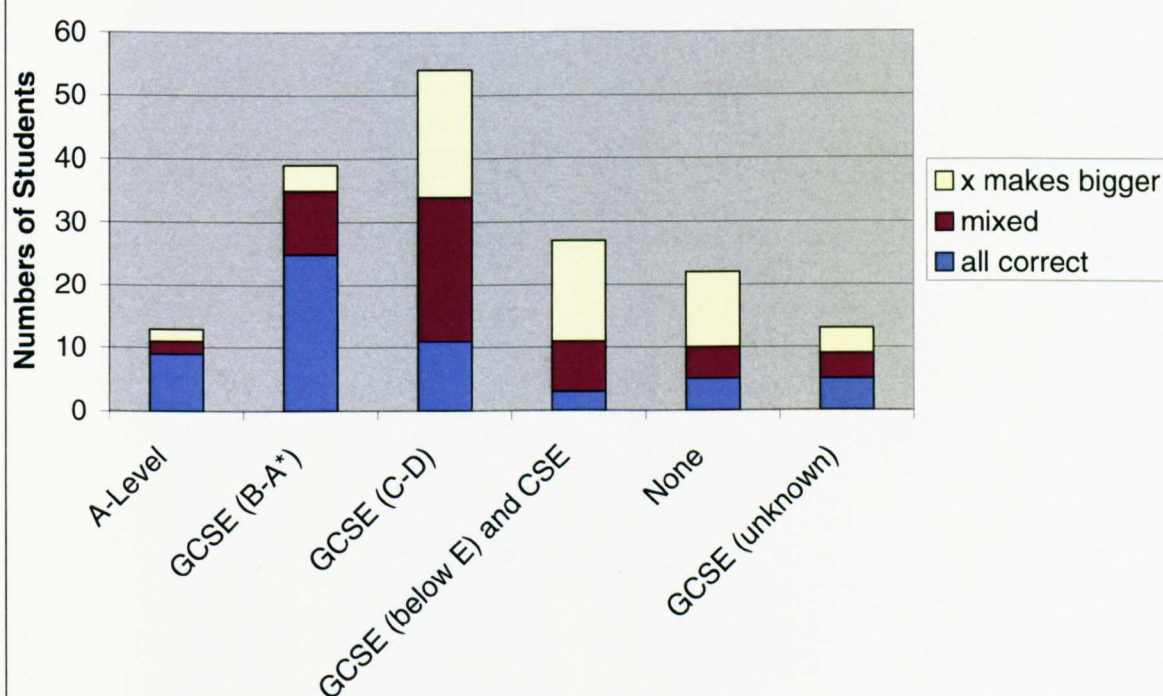
(7)  $6 \times 3$  or  $6 \div 3$ ,      (9)  $8 \times 0.4$  or  $8 \div 0.4$ ,      (11)  $0.5 \times 0.2$  or  $0.5 \div 0.2$ .

multiplication in the first case but only **32%** correctly selected all three operations. Although a larger proportion of adults than children selected all three operations correctly (**32%** adults versus **18%** 15 year olds), a larger proportion of adults also thought that multiplication always gave larger answers (**38%** adults versus **30%** 15 year olds) (Brown, 1981b, p.54).

Initially, I thought that this pattern was simply a function of the lack of stratification in the adult sample, that weaker students thought multiplication always gave a bigger answer and stronger students knew it did not. However, a quick overview of 2008 scripts showed a surprising pattern for one student. He or she apparently had an A in A-Level in maths and had answered all the questions in section A correctly except for the ones on multiplication makes bigger and the potato calculation which is discussed a little further on. In this case a stronger student apparently thought multiplication always gave a bigger answer. I decided to investigate further.

Issues about the reliability of qualification information and ranking have already been discussed indicating that formal correlation investigations were likely to be of little value. Instead, I considered all the home scripts from the 2006, 2007 and 2008 cohorts where the qualifications were clearly stated and unambiguous and clear attempts had been made at the multiplication makes bigger questions. This gave 133 scripts with clear GCSE grades, A-Level grades or CSEs, and another 35 with 'none' or GCSE without a grade which were a little more difficult to place. O-level results, Access and various others were not included at this stage. I separated the answers into three options: always thought multiplication was bigger (all x), was aware that multiplication was not always bigger but could not identify when this was (mixed), was correctly able to identify when multiplication was bigger and when it was not (correct). Figure 5.16 shows the results in terms of numbers of students.

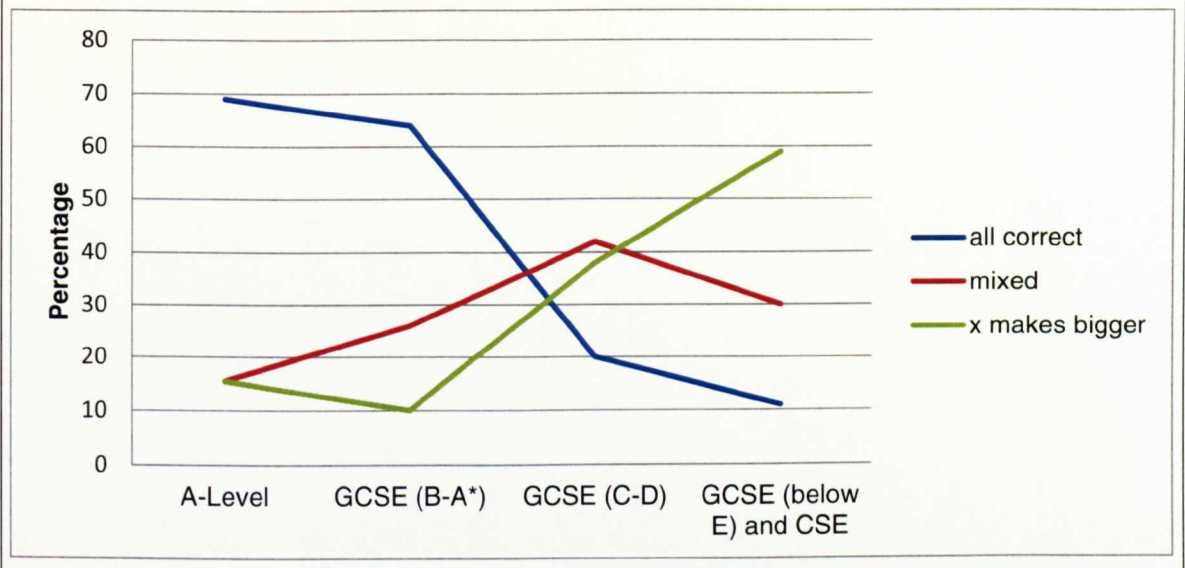
Figure 5.16: *Graph to show numbers of students giving different answers within each qualification subset. (Cohorts 2006, 2007 and 2008)*





Since there were more students at GCSE grade C than at any other qualification, it was more helpful to look at results in terms of the percentage of people at each grade who gave specific answers. The results are given in Figure 5.17.

Figure 5.17: *Graph to show percentages of students giving different answers with different qualifications. (Cohorts 2006, 2007 and 2008)*

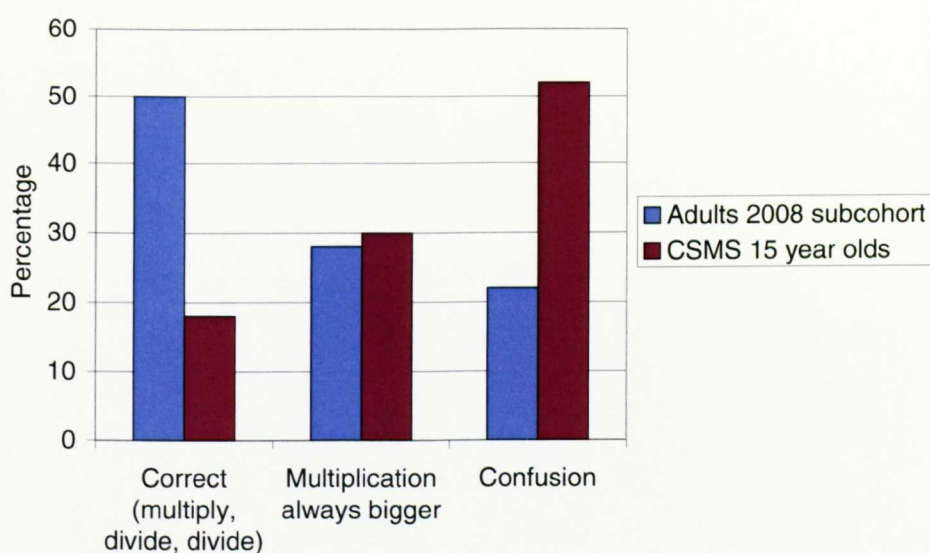


The graph shows an interesting pattern. The higher the level of qualification, the higher the proportion of people who are likely to identify correctly when multiplication makes things bigger and when it does not. The lower the level of qualification, the higher the proportion of people likely to over-generalise and assume that multiplication always gives a bigger answer.

Based on this information, the A-Level student response identified is not an anomaly, but is a relatively uncommon event. This performance raised two further questions. The first, 'How had they managed to get an A at A-level with this misconception and what effect might this have had on other performances?' indicates a line of enquiry that might raise some real implications for teaching. The second, 'Why had they done this?' might be answered by the consideration of individual behaviours that follows in the next chapter.

It was noted that the pattern of answers was slightly different when a sub-cohort of 33 students from 2008 were considered separately (Figure 5.18). This set of students only contained those intending to progress to either Primary Education or Biology based sciences and was less likely to contain large proportions of exceptionally weak or exceptionally strong students. An initial, somewhat

Figure 5.18: *Graph to show Percentages of Foundation Adults in 2008 sub-cohort and CSMS Children (Brown, 1981, p.34) identifying which calculation (multiplication or division) is bigger.*



simplistic interpretation of this data was that **28%** of this sub-cohort were over-generalising and using the rule 'multiplication is always bigger' in domains where this was not valid, that **22%** of the sub-cohort were aware that multiplication was not always bigger but were not sure how to decide when it was and that **50%** of the sub-cohort fully understood when multiplication was bigger and when it was not. The question that developed from this interpretation was whether any of the group who were now over-generalising had reverted or forgotten accommodations that they had previously made, whether they had reconstructed to fill the gaps in their knowledge (Duffin and Simpson, 2000; Karsenty, 2002) or whether there was

an element of dual processing (Stanovitch and West, 2000) and it was hoped that interview questions might help clarify this.

Consideration of the confidence data suggested that other factors also influenced answer selection. If people had simply applied the same rule to all three questions, it would have been expected that confidence in each of the answers would have been nearly the same. Similarly, if people fully understood when multiplication gave a bigger answer and when it did not, the confidence levels would have been expected to remain similar. This was not the case. **50%** of those who answered correctly indicated a drop in confidence as they progressed through the three questions and **66%** of those who answered multiplication for all three also indicated a confidence drop. Clearly other factors were involved.

All five students interviewed indicated some use of rules but the way they used them gives some further insight into the reasons for their confidence ratings.

Diane saw little difference between the questions:

*'This one again! -----  $8 \times 0.4$ '*

and explained that she was using a rule.

*'I would automatically just say the times ones, because I know that timesing them makes things bigger where as dividing things makes them smaller, but I am generally always wrong.'* (laughter)

Any lack in confidence was in herself, not in the questions. She explained any possible lapses with the later statement:

*'Things like this I probably would use a calculator for.'*

Her answer to a later question again indicated self doubt but trust in her rule.

*'I'm going to - see I don't know again because now I'm dividing it by 1.2 so I'm just drawn to saying the times one is bigger again'.*

Diane's follow-up questionnaire indicated a slight drop (very confident to confident) between the first and second question, but she incorrectly selected multiplication as the bigger for all three options. Interestingly, she correctly identified division for the third option in her initial questionnaire but with a drop to 'not very confident'.

.Brenda was also using a rule to over-generalise:

*'I'll immediately know if it's going to be a bigger answer it's going to be a times rather than a divide.'*

but paused to consider her rule again when the numbers became more complex:

*'So, I know that 0.4 is much smaller than 4 and 0.8 is much smaller than 8 so I think it's 8 times 4.'*

When the numbers became more complex still, there was evidence of a conflict and selection process (Harel *et al.*, 1994) but in the end she returned to her rule:

*'0.8 times 0.4, I'm second guessing myself there [laughter] cause when you multiply small numbers you get small numbers but I'm sure multiplying is bigger than dividing, so I'm going on that principle.'*

Following some discussion about her answers she concluded:

*'Ah.. right ... See I was just going on the assumption that multiply was bigger but yeah ... I think you shouldn't really, although you get used to ideas like times is bigger you shouldn't assume ....'*

and later recognised her own conflict:

*'I think I was starting to think like that cause looking at the two where you have got 0.4 and 0.8 I was.. I started thinking that these were small numbers and I was sort of second guessing it there.'*

Her incorrect answers in both questionnaires mirrored those in the interview. For the initial questionnaire she was very confident for all three questions but by the

follow-up questionnaire identified a drop to confident for the third question perhaps indicating an awareness of some conflict.

Christine had some memory of rules but not an understanding behind them. She correctly selected the division for  $8 \div 0.4$  and explained that she had a:

*'faint recollection of a teacher telling me once, when you divide by a decimal point it makes a bigger number than if you multiply, I might be wrong like.'*

Without any understanding she had some problems deciding on the third answer but used her own logic to decide:

*'Ooh I'm not to sure about that one, I've never had to work one of those out, As a guess I'd say the multiplication basically because it's the same, nought point and nought point so it's like  $8 \times 4$  again, you know what I mean.'*

Her answer to the later, more complex question again echoed her dilemma:

*'Oh no ... I would have said the multiplication one ... That's only because I always do it but it's that funny one where it's the low one where I get confused then on the other side it's the high one, I don't know why, I just do.'*

Consideration of Christine's questionnaires indicated that similar answers and confidence levels were mirrored in her written responses. She incorrectly selected the third multiplication as bigger in the initial questionnaire and correctly selected the third division as bigger in her follow-up response.

Ann appeared to have developed her own rules, could explain them and could extend them to the more complex question dividing by 1.2 later in the interview:



*'On Question A I think its 8 times 4, cause they are both large numbers. They are both whole numbers. On this one, on question B, I think it's 8 divided by 0.4. I'm not sure about that but I'm pretty sure if you are dividing by nought point anything it comes out as a bigger number ... I'll apply the same logic to question C and I'll go for 0.8 divided by 0.4.'*

She stated that she couldn't remember where her rules came from but it was interesting to note that she had incorrectly selected the multiplications for b and c in the initial questionnaire but had correctly selected division for b and c in the follow-up questionnaire. However, her questionnaire selection of 'not very confident' did not seem to fit with her competence in the interview.

Elaine was clearly highly anxious, lacking in confidence and using coping mechanisms (Leron and Hazzan, 1997) to try different strategies. Her incorrect responses (all multiplications) in her questionnaires were indicated as guesses but, since she had selected the guess option for most of the questions on the paper, this was of little discriminatory value. In the first instance she tried to calculate the answer:

*'8 x 4 is 32. If you divide 4 into there it gives you ... then if you half that one, hum I'm going to go for that one.'*

but had to guess the second answer when she had difficulty with the calculation:

*'It's a bit difficult, um nought point ... This is where I guess ... I think it ... might be that one.'*

However, her explanation showed she also appeared to be trying to use a rule:

*'Well I'm thinking because you multiply and you get a bigger answer'*

although her general anxiety and fear of fractions was confusing her further:

*'but there again it might not, I'm not very good with fractions ... My mind is*

*gone blank'*

leading to the following response for the third question:

*'That would completely flummox me it's a nought point'*

Although Elaine was clearly highly anxious selecting an anxiety score of 1 for both her questionnaires, some of her mixed approaches may also have been adopted by other students, not just those who were exceptionally anxious. The presence of written answers  $6 \times 3$  and  $6 \div 3$  on some papers indicated that others might be attempting to calculate answers and compare, although it is also possible that these were just used as confirmation for a decision already made. The increasing complexity of the calculations required, or an anxiety about decimals in general, might have lead to some of the recorded drops in confidence.

There was thought to be a possible difference between asking which calculation was bigger and in using these ideas indirectly in order to select which calculation to do from a word problem. In the former, the process of actually asking the question and recognition that a question might be asked in three different ways for a reason, might have triggered self-checking or dual processing for the instinctive answer (Fischbein, 1999; Leron and Hazzan, 2006; Stanovitch and West, 2000). In the latter there may have been more instinctive behaviour. For the A-level student discussed earlier, it was noted that the booklet was completed in 15 minutes suggesting little double checking.

Preliminary skim analysis of the 2008 sub-cohort scripts suggested that there might be some link between the understanding about whether or not multiplication gives a bigger answer and the successful completion of a question asking which calculation is required to find the cost of 0.52 kg potatoes at 78.4p per kg (chi squared  $p \approx 0.001$ ). The question responses are shown in Table 5.2.

It was later identified that this was not as significant for other progression routes and cohorts where weaker, more anxious students might have given slightly more random answers to some questions. In order to investigate further whether the

Table 5.2: <i>Analysis of answers for selecting operation and identifying when 'multiplying made bigger'</i>			
	Correct selection of when x is bigger.	Incorrect selection of when x is bigger	TOTAL
Correct calculation for Potato question	12	2	14
Incorrect calculation for Potato question	2	14	16
TOTAL	14	16	30

behaviour of those who had incorrectly answered the questions had been influenced by fear of decimals or difficulty in calculation, a small group of students were allowed calculators and asked to calculate the answer in class and then identify the calculation they should use. All eight who gave the correct calculation in the questionnaire, correctly calculated the answer in the class task. Six of these, then correctly identified the calculation as  $(78.4 \times 0.52)$ . The other two did the alternative, but also correct calculation  $(78.4 \div 100 \times 0.52)$ . Nine of those who had identified the incorrect calculation on the questionnaire or left blanks found the correct answer, with five identifying the first calculation and four identifying the alternative one. Since  $(78.4 \div 100 \times 0.52)$  was not available on the questionnaire this might have influenced the confused answers. One student, who had quoted 'no idea' on the questionnaire did a range of calculations and then incorrectly selected a division. The small numbers and the lack of returns from those with incorrect answers in the questionnaire limited the value of this exercise for the purpose of investigating 'multiplication is bigger' but did indicate that the ability to calculate might have had an influence on answers.

## 5.5 Conclusions

The adults in this study had a wide range of mathematical understandings, developed primarily from school experiences but consolidated, extended or adjusted through other experiences in adult living and the workplace, sometimes in unexpected ways. The majority of adult students were able to calculate simple percentages (Find A% of B) by using processes of halving, dividing by 10 or dividing by 5 and selected to use these processes successfully in a simple shopping question requiring a 5% reduction – a common real-life problem. In the follow-up questions, after some months on the Foundation Programme, and possibly in response to the influence of far more complex calculations which were difficult to solve without a calculator, some adults selected to use alternative calculation processes. For some this was more successful, for many less so.

For all other types of percentage calculations, facilities were much lower.

Students selecting one calculation in the initial questionnaire and swapping to another in the follow-up or interview were not uncommon, possibly sometimes affected by the semantics of the question (Kaput and West, 1994). Whilst the facilities for these questions were generally lower than for children and selection of calculation sometimes appeared to resemble a multiple choice problem, there was also evidence of estimating to confirm calculation choice.

Of the full home Foundation cohort only about a third were able to identify in the questionnaires when multiplication made things bigger and when it did not. Unlike the work on percentages, where there were clear indications of situating, of interactions with other real-life knowledge, the students interviewed were clearly using, rules they 'knew' from school. For those incorrectly over-generalising, the use of decimals confused and occasionally slowed down, but no memories of any

exceptions to the rule or discussions about it seemed to be triggered in memory. The people I interviewed were all over 25. I am aware that activities specifically designed to make these issues explicit have been incorporated into the former Numeracy Strategy, so it is possible that slightly younger adults might have demonstrated some benefits from this had I interviewed them.

## **5.6 Key Findings for Research Question One**

The first research question asked:

**What understanding do adult students have in certain mathematical areas and how does this compare with the understanding of some children?**

The following findings appear to emerge from the conclusions just given.

- 1) In some mathematical areas, the understandings of adults as tested through questionnaire completion appear to follow the same hierarchies in question facilities as those identified for children in the CSMS survey (Hart, 1981a). Closer consideration of methods shows that adults, like children, can often successfully use informal methods to solve proportional reasoning and some percentage calculation problems. In particular, the use of doubling and building, as identified in Brazilian street children (Carraher, 1991), in children prior to instruction (Lamon, 1994), and in children at different points in instruction (Hart, 1981b; Kaput and West, 1994), was in common use for foundation adults, particularly in initial questionnaires. However, a number of students transferred to more formal methods, particularly for percentage calculations later in the course (see Research Question 2).
- 2) In other areas and for many individual students, question facilities do not follow the same hierarchies as those identified for children fitting more with the confused mental map model described by Coben (2000).

- 3) Children had fairly similar success rates with different types of percentage problems. However, adults were far more successful with A% of B type problems than other types, possibly reflecting that these were the form most likely to have been met in shopping or other adult activities (Dowling, 1991).
- 4) Some of the mistakes made by adults appeared to be similar to those made by the children, suggesting that the thought processes might well be similar to those suggested by other researchers. Other mistakes were different and suggested the need for alternative explanations.
- 5) Like children, some adults used common overgeneralisations. A closer study of 'Multiplication makes bigger and division makes smaller' issues, showed that adults also seemed to follow the patterns suggested by Harel *et al.* (1994) for different number types.
- 6) Whilst some students did well in areas that have potentially been developed or consolidated since leaving school, some adults seemed to have higher success in areas such as simple algebra, which was unlikely to have been met since leaving school but presumably stayed in the mind better than some other topics.

In summary then, some adults' understanding, as indicated by their performance in mathematics tasks, appeared generally similar to that of the children in the CSMS study. However, for some students and in some areas results were more mixed. It appeared that success was often dependant on method selection and, therefore, the focus now needed to shift from what people were doing to why they were doing it. This is explored in the next chapter.

## **Chapter Six - Behaviours and Use of Strategies**

### **6.1 Introduction**

In the early stages of transcript analysis, it was suggested that consideration of behaviours and the use of strategies could best be achieved by looking at references to processes and asking the two questions: 'What are people doing?' and 'Why are they doing it?'. This question could also be considered in relation to questionnaire scripts, when such information could be found within them. In reality, the 'What' and the 'Why' were too heavily interlinked to be considered independently, but in the initial coding this separation helped to identify patterns. Many of the extracts seemed to resonate with the coping framework of Leron and Hazzan (1997) and fell readily into the principles of coping: making sense and meeting expectations. Others fitted with the idea of checking, and self-monitoring (Greeno *et al.*, 1999; Schoenfeld, 1992). A second set of extracts emerged around the general theme of cognitive errors. Some of the remaining excerpts seemed to fit more appropriately with the deeper consideration of maths topics and were transferred to that section. One final theme, 'calculators', was then identified from the remnants.

As each theme was considered and new ideas emerged from this consideration, questionnaires were revisited with 'new eyes' to identify additional data that might help support or contest these new insights.

### **6.2 Making Sense**

One of the first stages of problem solving, for expert and novice alike, is to make sense of the question (Leron and Hazzan, 1997; Schoenfeld, 1992). However, novices search for surface clues and then act on them quickly, whereas experts

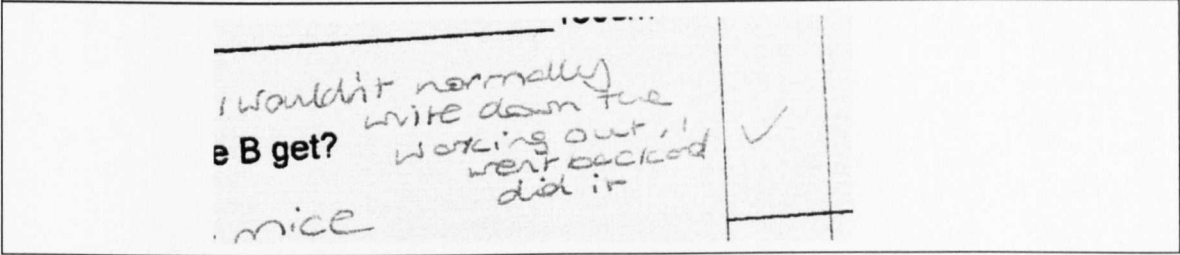
spend longer on initial reading and are more willing to change approaches if other ideas appeared more promising (Glaser, 1999; Schoenfeld, 1992). Hence, the strategies of the novice are more likely to cause them to follow red herrings or lead them astray (Leron and Hazzan, 1997). Both expert and novice might make use of instinctive and S1 processes (Stanovitch and West, 2000) where the search, find and solve are almost one action, but the expert is more likely to check these instinctive processes by using S2 processes (Fischbein, 1999; Kahneman, 2002).

All five students exhibited or referred to instinctive behaviour at some point. Some of these behaviours will be picked up later in the discussions on cognitive errors. Like the adults in Kirk and Ashcraft's (2001) study, Christine was aware that she had particular issues with trying to explain how her instinctive processes worked:

*'that's what I do I just I can jump straight to the answer but then I think they want the bit beforehand and it's trying to get it put.'*

The questionnaire extract shown in Figure 6.1 indicated that others also found

Figure 6.1: Extract about Retrospective Working



some difficulty with retrospective explanation. This may be why so many questionnaire answers showed little or no working.

One type of instinctive understanding was the ability to see the answer as a 'missing number' (Kaput and West, 1994). For the chocolate bar question some students seemed to be visualising the whole statement  $3 \times 4 = 12$  and were, therefore, able to identify the answer 3. The correct operation in the multiple



choice options might have been 12 divided by 4 and might have been selected because this too gave the answer 3 but did not represent the actual calculation the student did. Christine said:

*‘As soon as I saw the two numbers I knew the answer.’*

On that occasion she was correct, but instinctive use of number patterns appeared to sometimes lead others astray. When Ann was asked to explain why she had chosen two different methods for percentage questions (see Ann’s story, Section 7.3), she explained that because one question had used ‘round’ numbers she could see how to do it in her head but that when the numbers were more difficult she had needed to search for another way.

***Making Sense through number – Cost per item example***

Kaput and West’s (1994) studies of different types of proportion questions identified that semantics and number influenced strategy selection. Certain types of wording encouraged rate thinking so a question that gave the price for a number of items tended to scaffold people towards finding the price of one item first.

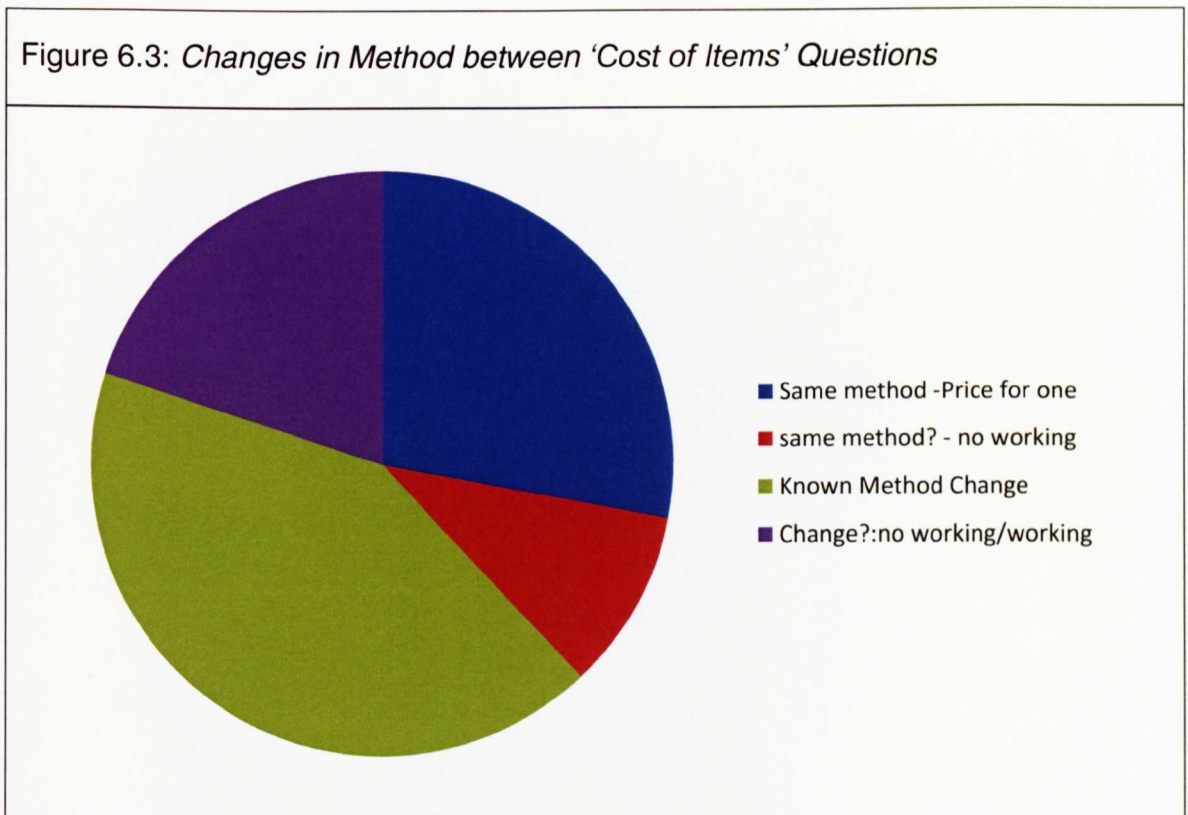
Figure 6.2 shows the two questions asked on cost of items.

Figure 6.2: <i>Question 18 and Question 6 (in second section) on cost of items.</i>		
<u>Question 18</u>	8 cards cost 48p. How much would 12 cost?	
- <u>Question 6</u>	6 cards cost 48p. How much would 11 cost?	

Using Kaput and West’s (1994) reasoning for Question 6, it would be expected that people would find the cost of one card, 8p, and multiply by 11 to give the total cost of 88p. As can be seen from Table 6.1, results from the scripts of 50 people who answered both questions tend to support Kaput and West’s (1994) reasoning. However, Question 18 produced some very different results.

Table 6.1: <i>Results from comparison of responses to 'cost of items' questions (2007, 50 scripts)</i>		
	8 cards cost 48p. How much would 12 cost?	6 cards cost 48p. How much would 11 cost?
6 x 12 or 8 x 11 shown	34 %	64 %
Just answer	26 %	28 %
Other correct method	12 %	0 %
<b>Total correct</b>	<b>72 %</b>	<b>92 %</b>

If method changes are tracked rather than the methods themselves, Figure 6.3, shows that something about question 18 caused people to answer it differently from Question 6.



Semantically, the questions are identical. The price for one in each case requires use of the same number bond,  $6 \times 8 = 48$ . Calculators were allowed for question 6 but not question 18. However, the majority of students opted not to use them. Additionally, personal classroom observations suggest that calculator use often

appears to discourage the writing of working, not the reverse. Therefore, it seems likely that it was the change in number that encouraged the change in method.

Pairs of answers were put together for closer comparison. A small selection of such extracts, (Students F – Y) chosen to illustrate some of the points discussed, is shown in Appendix 3c. In addition to the 'price for one' method, the connection between the numbers 8 and 12, allowed the use of various 'within' methods (Lamon, 1994), multiplying by the scalar 1.5, building  $48 + 24$  or composites of 24 (Students F, G and H). Some of these methods may have been instinctive, used automotiv processes and been completed successfully. Student J correctly found the answer for the first question but seemed unable to complete the 'cost for one' method used in the second question, supporting the idea that he/she had used an alternative instinctive method in the first question. However, for some students it almost appeared as if the availability of choice had reduced the probability of success rather than increasing it. One possibility was that the number ratio had become more 'accessible' (Kahneman, 2002) than the idea of the cost for one and methods using this ratio had 'hijacked' (Kahneman, 2002; Leron and Hazzan, 2006) notions of using other methods.

Looking at individual answers in Appendix 3c, even some of those who eventually selected to find the cost of one, temporarily explored alternative routes before rejecting them (Student K). For some students, the recognition of number links, 8 and 12 or 12 and 48 may have led people astray (red herrings, Leron and Hazzan, 1997) or somehow blocked the recognition that this question could be solved in another way, leading to somewhat convoluted and often incorrect solutions (Students L, M, N, P). Perhaps too much choice simply led to confusion and loss of meaning (Leron and Hazzan, 1997) and the panicked response of Student Q.

Whatever the detailed reasoning behind it, Ann's talk of the influence of 'round numbers' and her justification for changing the method in response to numbers:

*'Well I normally get to that first bit and think, oh they are harder numbers, there must be another way to do this, an easier way to do this.'*

was clearly enacted in the questionnaire responses for these two questions.

Whilst Ann is talking of finding, '*another way*' or using an alternative method, some of the examples from Students G, H,, L, N, P, Q, R, T (Appendix 3c) seem to go further and illustrate Greer's (1994) theory of 'nonconservation of operation', the lack of recognition that two problems which differ only in number can be solved by the same operations.

Lave (1988) argues that the maths practiced is a function of the practice itself. Following the interviews, it seems as if the methods used are also situated in the problem itself, not in the family of apparently similar problems. Problems that appear to be almost the same to an experienced mathematician, who is able to extract the calculation needed from the problem, and then put different sets of numbers into it, appear to be very different to the person who uses the numbers to select the calculation.

***Making sense through number – Or being led astray***

Two questions, one given in the 2006 questionnaire and the other in 2007 are shown in Figure 6.4. For the previous three questions in the questionnaire which

Figure 6.4: <i>Comparison of question from 2006 with question in 2007</i>	
<u>2006 question</u>	
Ring the number nearest in size to the answer of $71 \div 172$	
<u>2007 question</u>	
Which number is the nearest in size to the answer of $40 \div 200$	

were on place value, the facilities for both cohorts were fairly similar: within 5% of each other. For this question, the facilities were more than 20% apart with 23% of the 2007 cohort giving the reversed division answer 5. Evidence from elsewhere in the questionnaires indicated that those making this error were not confused about the non-commutativity of division and therefore parallel thought processes were in action (Fischbein, 1999). It appeared that the instant (S1) recognition (Stanovitch and West, 2000) that 40 divided exactly into 200 overruled a slower (S2) process which might have recognised that the reverse calculation was the one required. In line with Kahneman (2002) again, the more 'accessible' ideas had 'hijacked' the problem.

### ***Making sense through similarity with other problems***

Ann was influenced by the method she had just been working on (find A% of B) when she met a question requiring her to write a proportion as a percentage (see Ann's story in Section 7.3). Although she had successfully completed such a question in the past, on this occasion she tried to apply the method she had just been using. My response:

*'Careful! What does the question actually want?'*

Her response:

*'Oh it wants percentage ... even though it said what percentage is this before I even got as far as that I was thinking well which way round are they asking me to do this what am I supposed to be doing here? Without actually concentrating on there it's written what they are asking me.'*

Notice Ann's reference to searching when she asks:

*'What am I supposed to be doing here?'*

Some people might have been influenced by their expectation that a particular question was being asked. Evans (2000) recorded an unexpected drop in facility for a question asking what fraction was left after a 25% reduction. Many students gave the answer  $\frac{1}{4}$  rather than  $\frac{3}{4}$  but this did not mean that they did not understand percentages simply that they expected the question to ask something else.

Similarly, evidence from the interviews suggested that people answering the question on the chocolate bar (see Figure 6.5) might have been influenced, at least initially, by the usual 'real life' problem when one has a chocolate bar: the need to find the number of pieces.

Figure 6.5: *Questionnaire Question on Chocolate*

Question 1

*A bar of chocolate contains 12 squares.*

*There are three squares in a row.*

*How do you work out how many rows there are?*

(8 possible options were given)

Ann stated:

*'I looked at that to see, what are they asking me first of all? And then I thought yeah are they saying ... how many pieces are there and then I thought no they are asking ...the rows, they are not saying how many squares they are not asking that so I worked it out.'*

and from Diane:

*'This one ... No this one'*

Interviewer: *Right so you thought it was  $12 \times 3$  now you think it is was  $12/3$ .*

*Which are you going to settle for?*

Diane: (pause) *How many rows there are, yeah divided by.*

and she explains:

*'I was confused by this one asking it's broken into 12 like rows of chocolates and I was wondering how you eat chocolate and how many rows there are.'*

The interviewer sought clarification about what she had expected to be asked.

Diane: *How many squares of chocolate there are, not how many rows.*

Interviewer: *So it's possible if you were doing this in a hurry in a questionnaire, you might have done 12x3 ?*

Diane: *Yeah, even when I actually knew it was 12 divided by 3.*

At a later point the Diane's questionnaire was looked at.

Diane: (Laughter)

Interviewer: *That's what you just did, isn't it? That thing with the chocolate bar you have multiplied it.*

Elaine also gave the wrong answer at first:

Elaine: *It's C, 12 x 3.*

Interviewer: *12 x 3 Right, ok, what is the actual answer? If I asked you for the answer?*

Elaine: *12 squares, 3 squares -- there are 12 squares -- That would be 36 wouldn't it ?*

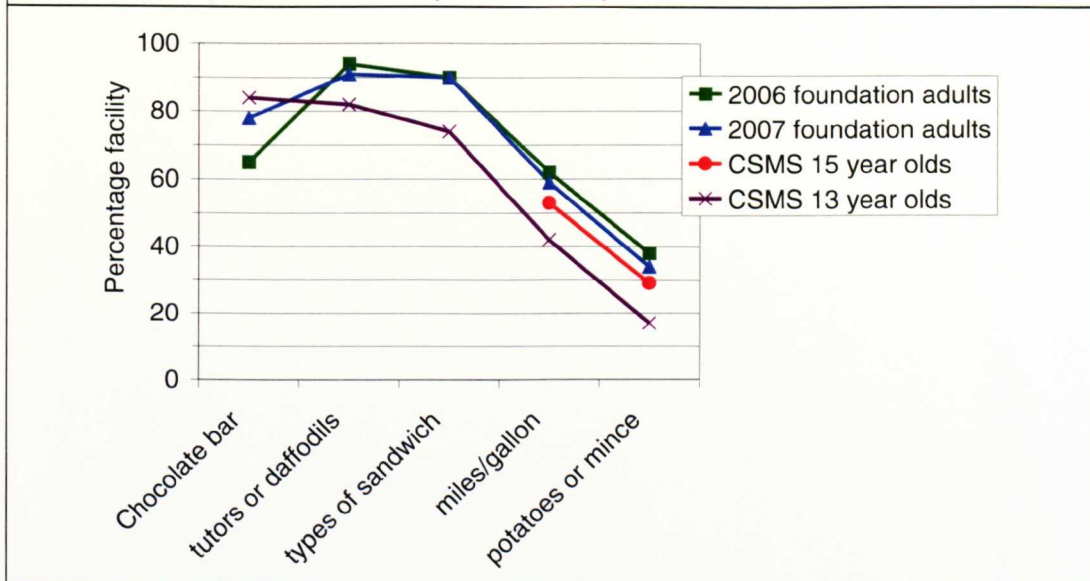
Interviewer: *It would.*

Elaine: *Oh yes, so there's 12 squares so that would be oh divide it wouldn't you - that into 12.*

Evidence from the questionnaires suggested that this issue was also a problem within the wider cohort. Comparison of multiple choice question facilities between

adult students and CSMS children are shown in Figure 6.6.

Figure 6.6: *Graph to show percentage facilities for questions on operation selection for adult foundation students and CSMS children (Brown, 1981a, p.41 and 55)*



This graph showed that the adult group had a slightly higher facility for all questions, with the exception of the one about chocolate given in Figure 6.5 which appeared to be an anomaly. It was not clear whether inverse operations and reverse thinking were included in the CSMS value which might have explained some of the differences, but it seemed that a number of students who successfully completed most of the questionnaire gave the incorrect answer to this question. Discussion at a BSLRM workshop presentation (Dodd, 2007b) suggested the possibility that the difference might have been caused by the presence of a diagram for the CSMS survey preventing possible confusion with other shape bars. No evidence was found to support this interpretation. Indeed, when Brenda was asked to explain what she was thinking she responded with:

*'What I'm trying to do is visualise it now so, visualise the chocolate bar.'*

Analysis of results from the 2007 cohort questionnaires indicated that the average confidence level for this question was 8.8 which, together with two other questions,



was the fourth highest out of 34 questions. However, the facility for this question at 78 was only 13<sup>th</sup> from the top, indicating that a number of people who felt confident that they had answered the question correctly had not. Further, it was noted that the facility for this question had risen from 64 in the previous year when students had not been allowed to look back and check their work.

Taken together, all this evidence suggested that the lower question facility was the result of using a real-life situation to pose a non-real-life problem. The semantic influences in the question encouraged the wrong answer. In effect, the question was testing students' care in reading questions not in mathematical understanding and could therefore almost be described as a 'trick question'.

### **6.3 Meeting Expectations**

In Section 2.4 it was noted that problem solving required an understanding of purpose or desirable outcome. Solutions needed to meet the expectations of an 'authoritative other' (Leron and Hazzan, 1997) whether that 'other' was real or 'virtual' as in the sense of an imaginary arbiter of academic practice. Further, it was suggested that people might sometimes measure their outcomes against their own beliefs or expectations of what problem solving should be and that these beliefs might undergo change in response to induction into the academic 'Community of Practice' (Lave and Wenger, 1999).

Real 'others' might include the person marking a question paper, a teacher from the past or someone physically present and potentially interacting with the solution process. All three of these 'others' appeared to influence the decisions of those in the interviews, but the influence of past teachers is visited in the next chapter.

## ***Meeting the expectations of the Question Writer***

Patches of communication appeared in a variety of formats within questionnaire scripts, varying from apologies, 'I'm sorry', to explanations, 'I haven't a clue' to smarticons (mainly sad faces) indicating a recognition that a person and not some marking machine would be looking at the work. The extract previously shown in Figure 6.1 indicated that some people were more aware of the purpose of the questionnaires and consciously trying to provide the information that might please the researcher. Christine noted:

*'I knew the answer but I knew you wanted the working out.'*

Both Ann and Christine seemed to attribute the question writer with a desire to catch them out. Both mentioned at different points that they thought something might have been a 'trick question' and Ann mentions that she checked an answer because she was 'suspicious'. Ann expected questions to be difficult:

*'then I thought 'no it can't be as simple as putting 3/8ths''*

Diane seems to imply that the writers deliberately set out to make things impossible:

*'I was thinking 'she can't be serious'.'*

Gardner (1999) noted that many assessments are designed to focus on what is not known, the weaknesses, rather than the strengths and it might be experience of this in previous schooling that had guided their thinking. Note that Ann adjusted her behaviour to meet the perceived expectations of the 'authoritative other' (Leron and Hazzan, 1997) in this case the question writer. Her belief that a question should be difficult caused her to reject her correct answer.

It seemed that people had begun to identify the expectations of academic practice and the need to adjust behaviours to meet different expectations (Maier, 1991;

Roth, 1999). Referring to the questionnaire, Ann explained that:

*'it depends what I was doing it for, if I was doing it for an exam like this, I would do ... but if I was just sort of out and about I would ...'*

Diane connected me with the questionnaire writer, and, therefore, the need to meet my expectations, when she discussed whether or not to give both possible solutions in a multiple choice question:

*'I wasn't sure whether you wanted it to be what it was or what it could be as well.'*

However, she explained that by the time of her second questionnaire she did not face that dilemma:

*'On this one (second questionnaire) I thought, oh well there is no point doing 25 of 100, I already know that it's a quarter where as on this one (first questionnaire) I was like, it is a quarter but which one should I put?'*

She had now been partially inducted into the community of academic practice (Lave and Wenger, 1999) and developed her own beliefs about acceptable academic outcomes.

Her decisions were mirrored by others in the questionnaire. In questions 2,3,4 and 6 there were two possible correct answers. Many of those who selected both answers in the first questionnaire selected just one answer for all the questions in the follow-up questionnaire, perhaps reflecting new beliefs about the expectations of academic practice. A few, all on biology routes, reversed the trend and went from one to two solutions.

A number of students swapped from more spontaneous methods in the initial questionnaire to more formal methods in the follow-up. There was a possibility that closeness to January exams had influenced people's selection of strategies

so the questionnaires for the following cohort were administered in April.

However, the issue of changing method remained, suggesting that this also might reflect new beliefs about expectations (Maier, 1991; Roth, 1999). For the former answers were sometimes given with no working. For the latter, many people gave more formal working but did not complete the calculation. It appeared the shift for some people had moved from emphasis on the right answer to emphasis on the right method and people did not calculate the answer, possibly because they were not allowed a calculator. Clearly, people were not only adjusting their method for different perceived outcomes (Maier, 1991; Roth, 1999) but were also perceiving tasks differently as they became enculturised into the expectations of a university.

### ***Meeting the expectations of the person present***

Leron and Hazzan (1997) described how a person (Johnny) who was experiencing loss of meaning, felt pressured to provide 'an answer, any answer' to please the 'authoritative other'. My presence in the interview may well have created pressure and interfered with thought processes, but seems to have had an opposite effect for Elaine to that observed with Johnny. At different points in the interview she willingly admitted that she did not have an answer:

*'My mind is gone blank ... it would totally flummox me ... I give up.'*

However, interestingly her other comments suggested that she felt pressured to provide 'an answer, any answer' (Leron and Hazzan, 1997), in the questionnaire or exams:

*'£16 its probably totally wrong I'm dividing 5 into 20 and it's totally wrong, and if I was sitting in an exam I would put about £16.'*

or, referring to a questionnaire question:

*'I thought I can't leave it blank.'*

There were certainly occasions when all five interviewees were attempting to use my presence as 'authoritative other' to help find sense or ratify their ideas. Elaine for example on meeting a percentage question, checked before proceeding:

*'Do you multiply by 100?'*

## **6.4 Checking and Self-monitoring**

It has been noted (Glaser, 1999; Greeno *et al.*, 1999; Leron and Hazzan, 1997; Schoenfeld, 1992) that one of the distinctions between novice and expert is the amount of checking and self-monitoring that takes place during the problem solving process. It was evident from questionnaires that some level of checking was taking place on most papers because sometimes the first answer given had been changed, usually, but not always, when the first answer was incorrect. Sometimes the working simply stopped as if the answer had been calculated in the head, rejected and it was not worth continuing (see Student T, Appendix 3c). Sometimes it looked as if students had elected to change method part of the way through, perhaps indicating self-monitoring (Greeno *et al.*, 1999; Schoenfeld, 1992), (see for example Student K, Appendix 3c).

All five interview students indicated the use of checking as a final stage of confirmation that an answer was correct and indicated their belief that they could identify incorrect answers. Brenda said:

*'That's the funny thing, with working it out I'm actually quite slow to get to the number, but I'd know which was the wrong kind of number, like if I'd got something like 100 I'd know it was bad.'*

Ann said:

*'so I'd want 6/8ths of a packet - No I wouldn't cause that's more than a half. - I don't know.'*

Ann's response to this problem could be seen as simply giving up, recognising that her answer was wrong but that she didn't know any other way to do it.

Alternatively, it could be seen as a recognition of conflict between her expected answer and her calculated one, with her statement, '*I don't know*' as a hidden request to me as 'authoritative other' (Leron and Hazzan, 1997) to tell her which idea was correct.

Christine identified that she frequently made instinctive, S1 type errors (Stanovitch and West, 2000):

*'I don't double check. That is my biggest fault with them.'*

Hence, her response to a wrong answer was to check her working:

*'When it comes out wrong I'm thinking, 'where have I gone wrong?''*

It was clear that people sometimes had an idea of the sort of answer they were expecting. For example, Diane said:

*'I kind of already knew that's how it would look.'*

At different points in the interviews, students indicated that they were checking answers, rejecting wrong answers and then trying something else, fitting with Benn's (1997) observation that adults often continued working until a reasonable answer was obtained. Diane said:

*'I kinda thought It can't be 36, it definitely isn't, because it would have to be closer to 300 mice because 300 is like half and 42 do you get what I mean, it can't be 36, it definitely isn't so then I did it the other way around I think.'*

and explaining further:

*'It was more like a trial and error thing.'*

Brenda said:

*'I think it should be 12 divided by 3 because 12 times 3 obviously gives too big a number.'*

which suggests either that she thought of  $12 \times 3$  first and quickly rejected it because it was too big or she specifically made a choice between two possible calculations.

The difficulty of course is in identifying what is a reasonable number. For Christine expertise in checking is successfully linked with real life experience in estimating (Maier, 1991; Roth, 1999) to work towards one answer:

*'but I knew roughly the answer because I knew it had to be less than 400 cause 43 is nearly 50%.'*

or later:

*'because if I need less, then it can't be  $\frac{3}{4}$  if I only need half a box.'*

However, later on she identified that she knew the importance of checking but might not always be able to tell what was reasonable unless it was way off:

*'It's very silly cause we all rely on calculators now, but if you don't know generally what it's gonna be - if you get a really really big number you're gonna think well that's wrong.'*

Schoenfeld (1992) described how experts frequently tried a number of alternative ideas before selecting the most appropriate route for problem solution. In one sense then, the excerpt below shows the use of some expertise:

Interviewer: What would you put in the calculator?

Diane: I don't know. I would try all sorts of different things until I got what was right, but I think it's, Is it  $24/800$ ? (laughter)

Diane clearly had gaps in her knowledge base (Schoenfeld, 1992), and, therefore, had poor mathematical understanding. However, her method on this occasion

could be argued to have been a viable one, even if it was not efficient. It is only if she were not able to recognise a reasonable answer, or decide between two alternative reasonable answers, that her method would break down. Her earlier statement of a 'trial and error' method suggests that some of her other calculations could be interpreted in this way too. I previously suggested that Brenda might be selecting between two options when she selected  $12 \div 3$  rather than  $12 \times 3$  because  $12 \times 3$  was too big. Could it be argued she also used a trial and error method?

This idea of deliberate use of trial and error has some obvious implications.

Firstly, if this method appears to provide satisfactory solutions, using Lithner's (2008) argument there is little motivation for a student to find another way or to try and understand more deeply. Secondly, the apparent success has the potential to mislead others. The assumption that success reflects mathematical understanding and/or the use of logical reasoning similar to that used by a tutor, might well interfere with interpretations of where a student is on their learning journey. Successful learning interactions need to be targeted within a student's Zone of Proximal Development (Vygotsky, 1978). A lesson designed to build on existing understanding is unlikely to be successful if the understanding is not there to build on. Thirdly, the trial and error method breaks down if people are unable to identify when an answer is reasonable, suggesting one possible reason why some adults seem to experience some major disempowerment with Chemistry.

In summary then, there was some variation in the ways that checking was used and its importance within the process of obtaining an answer. Whilst in some cases it was used simply to confirm an answer, in others it was a more integral part of the process and used either as a rejection tool in a trial and error process or to select an answer from a range of possibilities.



## 6.5 Cognitive errors

### ***Incorrect recall of number bonds***

The rapid recall and use of chunks of information, including number facts, is a valuable tool in problem solving (Glaser, 1999; Greeno *et al.*, 1999; Schoenfeld, 1992). However, it has also been noted that certain number bonds can be more readily recalled than others and that storage and retrieval may be effected by associative strength and/or interference from other number facts (Cowan, 2003; LeFevre *et al.*, 2003). Alongside the increase in potential interference from number facts accumulated through work and home, adults' 'reservoir of experience' (Knowles, 1980) may contain flawed information or mis-remembered facts (Schoenfeld, 1992) sometimes generated by previous mistakes consolidated through use. Alongside errors caused by incorrect retrieval, for example confusing two multiplication facts ( $3 \times 8 = 32$ ), so-called close miss-errors sometimes occur where an answer is one or two away from the correct number (Cowan, 2003). Both kinds of errors were identified within the questionnaires.

### ***Interchanging $\frac{2}{3}$ of 12 or $\frac{3}{4}$ of 12***

When analysing the responses to proportion questions on snakes it was noted that **8%** correctly answered a question requiring a multiplication of 1.5 but incorrectly gave an answer of 9 for a question requiring the finding of  $\frac{2}{3}$  of 12. Similarly, **20%** of people gave the answer to  $\frac{3}{4}$  of 12 as 8 in a later question. Closer inspection of responses showed that some students seemed to be swapping between answers, sometimes selecting one and sometimes the other both within and between questionnaires. This resonated with the findings of LeFevre *et al.* (2003) that adults used multiple strategies and did not always rely on retrieval of stored facts.

It also suggested that dual processing (Stanovitch and West, 2000) was occurring.

To investigate further, two questions ( $\frac{2}{3}$  of 12 and  $\frac{3}{4}$  of 12 ) were given at different points in the interviews. All five students correctly calculated the answers in this situation by working out one third or one quarter first and then multiplying, even though two of them had given at least one incorrect answer in their questionnaires.

Ann was asked whether she thought she would always work out her answers in the same way as in the interview or whether she would just write down the answer. She stated:

*'I think I would have leant towards working it out again.'*

In fact, she had incorrectly given 8 as  $\frac{3}{4}$  of 12 in both her questionnaires.

Diane incorrectly gave 9 as  $\frac{2}{3}$  of 12 but correctly gave 9 as  $\frac{3}{4}$  of 12 in her initial questionnaire. She correctly gave both answers in the next questionnaire. When she went through her answers in the interview she gave some further information:

Interviewer: *Two thirds so you wanted two thirds of 12.*

Diane: *Yeah so I put 9.*

Interviewer: *Which is three quarters of 12.*

Diane: *(laughter) I think I actually put 9 there first, and then I thought again and thought 'oh no it's 8 I think' so I scribbled it out.*

Elaine also linked the questions together in her mind:

*'Oh, we're back to this one!'*

The completion of the task in an interview session may have raised the confidence threshold (LeFevre *et al.*, 2003) so that people were either less willing to rely on direct recall, or more likely to check answers derived instinctively with S2 processes (Stanovitch and West, 2000). Alternatively, the need for explanation

may have encouraged people to report non-instinctive processes even if parallel or dual processing (Fischbein, 1999; Stanovitch and West, 2000 ) was taking place (Kirk and Ashcraft, 2001). Additionally, requiring the calculation in isolation rather than as an intermediate in another problem might have caused more focus and triggered the need for greater checking (Fischbein, 1999).

### ***Close-miss errors***

Two of the five students interviewed identified occasions when they wrote answers one more or one less than they meant to, so called close-miss errors (Cowan, 2003) but realised their mistake when looking through again. Elaine stated:

*'I just thought 24 out of 300. It should have been 72 not 73.'*

Christine stated:

*'I think I've done  $5 \times 3$  is 16, that's so stupid and I can't even imagine why I would do that.'*

However, the later comment of Christine suggested that this was actually a more frequent occurrence for her:

*'I do basic calculations wrong sometimes, you know when I'm just writing away, I have been known to just put .. strange answers, either 1 less or 1 more than what the answer is, and people just look at me and go, but you can do them.'*

Whilst it is clear that locating errors led to individual frustration for Christine, the latter part of her statement, where others have linked her performance to her understanding (*but you can do that*) is particularly interesting. If the presence of such errors has the potential to cause misdiagnosis of understanding by student or teacher, failure to recognise such errors may result in attributing answers to lack of understanding and focus attention on concepts which have already been grasped,

leading to a drop in individual confidence. The reverse is also possible in that the combination of a misunderstanding and a careless error together may result in the 'correct' answer. Making such errors explicit to students might make the instruction to 'check your answers' much more targeted and help raise confidence. As Leron and Hazzan (2006, p.124) state:

If analyzing typical S1/S2 pitfalls became an inherent part of students' problem solving sessions, they might become more successful problem solvers and decisions makers.

### ***Losing track***

When considering the interview transcripts, the phrase '*lost track*' or '*I got lost here*' were found to keep appearing (see for example, Ann's comments on calculating  $\frac{1}{2}$  of a half packet). Similarly, this comment was occasionally to be found on scripts, usually following a pile of working which had been abandoned. Baddeley (1986) suggested that tasks are completed using a working memory, which not only stores information but also allocates cognitive resources to different aspects of task activity. Just and Carpenter (1992) noted the capacity of the working memory is limited and near this limit there appeared to be a trading between storage and processing, causing older information to be forgotten or displaced, a possible explanation for 'losing track'.

This reasoning might also explain why there were many examples where drops in facility seem to be associated with an increase in complexity of calculation. This difference was particularly noticeable between questions requiring just one stage of working and those requiring more, for example moving from the picture question that required 5% to the later question that required 6%. In a similar way, Hart (1981b) noted that with more complex tasks some of the children in the CSMS

survey gave an incorrect answer because they were unable to recall all the intermediate values.

This seems to be mainly of a problem of memory storage, taking the wrong sub-answer to work with from the pile of stored sub-answers or finding the earliest sub-answers have been emptied out (Just and Carpenter, 1992). Differences in questions facilities when an intermediate answer was required and when it was not, have already been referred to in other sections (see for example ratio) and previously attributed to scaffolding (Kaput and West, 1994). The discussion here also suggests that the forced storage of an intermediate answer might have been advantageous, not only in ensuring the answer is not lost but also in releasing additional cognitive resources to work on other areas of the problem.

The longwinded nature of this process might itself alert an experienced student to the need to check the answer, although the answer itself might not. However, for some, low confidence students, the result of a long calculation may be the belief that it 'cannot possibly be that complicated', (in contrast to Christine's idea in Section 6.3 that it 'could not be that simple') resulting in low trust in answers or in extremes in crossing out of correct answers. Examples of both of these activities could be found amongst questionnaires.

For me, the more interesting process revolves round this notion of track or route. Whilst much of the literature discussed earlier uses language from a more psychology based discourse and refers to concepts such as 'cognitive economy' (Cowan, 2003), use of this everyday language creates a picture which generates some additional ideas. The fact that it was possible to 'lose track' seems to imply that the track might not be clearly marked. If the definition of intuitive and S1 processes as fast, automatic, unconscious and effortless (Leron and Hazzan,

2006) can be applied to the method selection process as well as the calculation, people may not be explicitly aware of what they are doing and where they are going. This makes them vulnerable to interference from other factors. In effect, taking their eyes off the route, using their brain to do other calculations, causes them to lose the way. Elaine also appeared to use the journey analogue:

*'Sometimes I get an idea and I sort of get lost in it and I think I've taken a wrong turning here somewhere.'*

Ann looked back to her attempt to calculate fractions of a packet and also appeared to be implying a lost route:

*'I was thinking what am I multiplying here? I'd lost track of it a little.'*

A more experienced mathematician might be able to recall number facts, chunk information and carry out side calculations in an automatic way (see Greeno *et al.*, 1999) so is less likely to lose the route. Further, an experienced mathematician might have been able to extract the route from the problem, have an explicit map they are following, so that they can continue on the journey even if their mind is temporarily side-tracked to work out something on the way. In contrast, an inexperienced student may not have deliberately selected a route or even be explicitly aware that they were following one. Hence, a teaching process which attempts to transfer methods to solve another problem is unlikely to be successful.

## **6.6 Calculator Use**

This research did not set out specifically to study calculator use; indeed the use of calculators was deliberately prohibited in the 2006 questionnaires and Part A of the 2007 questionnaires. However, even in their absence they appeared to have potentially influenced behaviours and were mentioned by all five students in the interview. For this reason some discussion on the role of calculators is included

here within the context of problem solving and aiding cognitive economy.

In the comparison of initial and follow-up questionnaires it has been noted that a number of people wrote out the full working for a problem but were unable to complete the calculation because they were not allowed a calculator. The use of the calculator appeared to have become more legitimised or indeed expected (Maier, 1991) and has been argued to be a part of induction into the 'Community of Academic Practice' (Lave and Wenger, 1999). From a more practical viewpoint, it may have simply been that the types of calculations met on the course, particularly in the sciences, used numbers which would not have been readily amenable to mental methods.

If calculators are seen as simply a more reliable tool for calculation than the use of direct retrieval or standardised multiplication and division sums, the calculator could be argued to only affect accuracy of answers. Perfectionist aspirations (Siegler, 1988) and raised confidence thresholds (LeFevre *et al.*, 2003; Siegler, 1988) with the different perceptions of task might indicate why people moved towards more calculator methods in later questionnaires. As Ann said:

*'I think anything a little bit more difficult than that I would probably double check on it with a calculator to be honest.'*

If, however, the calculator is recognised as a tool to aid cognitive economy (Cowan, 2003), the decision not to allow calculators may have caused more students to 'lose track'. The use or not of the calculator and its potential to generate an answer more rapidly, might also influence the ability of students to check reasonableness of answers (Benn, 1997) or even to select between methods for those using a trial and error type approach (see Section 6.4).

Diane had clearly embraced calculator use:

*'Things like this I probably would use a calculator for.'*

and later:

*'I trust my calculator more than I trust my brain.'*

However, Christine had a more mixed relationship with the calculator probably stemming from her school experience:

*'See at school we didn't have a calculator, I had to use my head.'*

On two occasions she directly blamed her calculator for leading her astray:

*'I would have been right but because my calculator was there I thought, alright I'll put it in.'*

and later:

*'No idea whatsoever (10 sec pause) it's cause I did it on the calculator.'*

However, she accepted that calculators were here to stay:

*'we all rely on calculators now.'*

and that she would be making use of them:

*'No the thing is I've got a calculator now and that does it for me.'*

It needs to be remembered that these interviews took place after teaching had finished and so the use of the word 'now' might be interpreted to mean that she had developed her new calculator awareness as a consequence of Foundation learning. This final extract from Elaine where she identified that she would prefer to use a longhand method for finding the mean and standard deviation (involving quite complex intermediate calculations following a formula and using her calculator with BODMAS rules) rather than a stored memory function, perhaps highlights the sometimes contradictory actions of adults and calculators:

*'There's a quick way to do it on your calculator. I can't do it. I can't remember so I've drawn the square with the x and the x-x and the x- x*



*squared on my paper because that was the only way and ... said to me, "why are you doing it longhand sort of thing?" I said "because it is the only way I can do it" and she said , "you are so funny, you can remember the long way but you can't remember", ... That's like double Dutch, I've put it in my calculator and I've got these n's and little x's with lines on it and I thought "What are they? What are they for? Go back to your boxes". I'm alright in my boxes.'*

## **6.7 Conclusions**

A wealth of ideas on behaviours and strategies have emerged from consideration of the interview transcripts and revisiting of questionnaires in the light of this new information. Central to the progress of this research has been the emerging theme that adults appear to answer mathematics problems according to a complex range of interacting criteria, many of which are hidden from the adults themselves.

The process of problem solving within interviews, and by inference within questionnaires, appeared to be in part a method of decision making and selection, some of which was instinctive and subconscious and some of which was more deliberate or explicit to the student (Cowan, 2003; Fischbein, 1999; Kahneman, 2002; Kaput and West, 1994; Lefevre *et al.*, 2003; Leron and Hazzan, 1997, 2006; Schoenfeld, 1992; Siegler, 1988). Numerous examples were found during analysis where adults switched methods between questionnaires and interviews. In particular a number of students swapped from more spontaneous methods in the initial questionnaire to more formal methods in the follow-up. For both questionnaires, the identical rubric required an answer. For the former this was nearly always given, sometimes with no working. For the latter, many people gave working but were unable to complete the calculation because they were not

allowed a calculator. The use of the calculator appeared to have become more legitimised or indeed expected. Clearly, people were not only adjusting their method for different perceived outcomes (Maier, 1991) but were also perceiving tasks differently as they became enculturised into the expectations of university life, hence Ann's comment, 'In an exam like this' .

With different answers, different methods and sometimes different levels of success for the same question asked on different occasions, it is clear that knowing in one context does not necessarily equate with knowing in another, which of course fits with one of the key tenants of the exponents of situated learning (Lave and Wenger, 1999). The understanding exhibited in one situation could be argued to be a snapshot or subset of all the understandings in all the situations. Discussions on expertise have highlighted the need for meta-cognitive skills, particularly self-monitoring and self-regulation (Greeno *et al.*, 1999) and for adaptive expertise (Baroody, 2003) which allows people to understand the relative advantages of alternative methods and select the most appropriate or efficient one for different occasions. In practice the goal of foundation teaching is to gain understanding in the maths to be met during degree study and as such, some understandings carry more value than others.

It was noted that many students seemed to have an intuitive understanding in some areas and solved problems instinctively (Fischbein, 1999; Kahneman, 2002). However, looking back at their work they sometimes found difficulty explaining or justifying what they had done. Kirk and Ashcraft (2001) noted that the need to explain methodology or show working sometimes caused people to reject intuitive methods in favour of slower methods that could be explained. The very complex and highly unlikely explanation provided by Elaine feeding snakes and eels is a

good example of this. This was further complicated by the belief that these methods were not legitimate (Duffin and Simpson, 2000) or perhaps less academic given their aspirations for a university persona.

Dual or parallel processing (Fischbein, 1999; Kahneman, 2002; Leron and Hazzan, 2006; Stanovitch and West, 2000) was clearly in operation for some people. A good example of this was given by the confusion between  $\frac{2}{3}$  of 12 and  $\frac{3}{4}$  of 12. When this was the main focus and the calculation effectively slowed down by the need to report, all the interviewees divided then multiplied and produced the correct answer. However, when this was met within a ratio question where the focus was on different areas, some people swapped answers as if the learned results were stored in the same place and occasionally interchanged, the closeness of the two answers, 8 and 9 failing to trigger any S2 (Stanovitch and West, 2000) processes.

Perhaps one of the most interesting findings for future teaching is the discovery that, for some adults, the methods used were a function of the numbers in the question, not of the family of similar questions to which this question belonged. Whilst Greer (1994) was discussing this phenomenon for children as long ago as 1986, I believe that this idea is not embedded in adult literature and certainly not in common usage amongst the foundation lecturer population. For example, the 2009 Foundation Network Conference considered the use of software packages which interchanged number, order and context to provide 'equivalent questions' so that people could be tested in large groups without issues of copying.

## 6.8 Key Findings for Research Question Two

The second research question asked:

**To what extent does the mathematical behaviour of adults mirror their understanding?**

The following findings appear to emerge from the conclusions just given.

1) As expected from considering research by others, adult problem solving behaviour is influenced by context and task environment. Perceptions of these change as adults become encultured into the University Community of Practice. Some students move from spontaneous methods to more formal methods as they progress through the year.

2) People could be argued to **know** in one situation but **not know** in another.

The understanding exhibited in one situation is a subset or snapshot of all the understandings in all situations.

- A person may be able to confidently provide a correct answer in one task, but have insufficient skills or understanding to produce a correct answer using an appropriate method in an alternative task within a different context or environment on another occasion.

3) The methods selected by an adult for a particular mathematics problem are a subset of the possible methods and solutions that might be available to them.

- Adults sometimes selected different methods for the same written question in each of the questionnaires and interview.
- Adults are sometimes explicitly aware that they need to select different processes for different outcomes (or for outcomes perceived as different).
- The perception that certain algorithms or techniques are more highly valued or provide more acceptable outcomes within university settings sometimes

influences practice.

- 4) Adults sometimes use intuitive or instinctive methods which they have difficulty explaining or justifying. Sometimes, these are more successful than other methods and sometimes, less so.
  - A requirement to explain methodology or show working may cause a student to abandon that method of working.
  - The process of slowing down can interfere with the intuitive process but may also provide an opportunity for alternative checking systems to come into play, or for dual processing to take place.
- 5) The method an adult selects to solve mathematics problems may be a function of the numbers within the problem. Adults may elect to use different methods for solution of families of problems which differ only in the numbers involved.
  - The recognition, often instinctively, of a connection between numbers may lead people towards a particular method
  - Sometimes an answer can be seen, not calculated, thinking forwards rather than backwards and filling in the missing space.
  - Sometimes a conscious decision is made that more complex numbers need to be dealt with by calculator or need justifying or belong in an alternative context where different methods are valued.
  - Sometimes an internal trial and error or multiple choice process takes place which may be so rapid, people are not aware they are doing it.
- 6) Adults sometimes exhibit elements of behaviour associated with the more expert problems solvers when they carry out a number of CAT cycles in their work and routinely check for reasonableness. This may present issues if:
  - an overgeneralisation such as 'Multiplication makes bigger' corrupts the recognition of reasonableness.

- the underlying theory within the problem is too unfamiliar to allow an understanding of what a reasonable answer would be.
- 7) Sometimes an adult selects the method to use to solve a problem by first identifying an answer, checking for reasonableness and then trying another if required. The first answer may be generated by:
- doing the calculation thought to be most likely,
  - completing some instinctive calculation suggested by the numbers,
  - using the numbers in the order in which they appear in the question,
  - trying every combination and selecting which seems best.
- 8) In problems requiring a number of stages, some students can successfully follow an instinctive linear process to the solution but lose track or fail if the numbers become too complex and calculations less automotive or instinctive.
- 9) Sometimes adults could be argued to understand a problem in that they can identify the process, extract the maths and select an appropriate calculation method but this understanding is masked by errors.
- Competing associations from home and work may lead to retrieval errors.
  - Misattributing incorrect answers to lack of understanding rather than calculation error can lead to focusing attention on the wrong area for repair and causing frustration or feelings of failure.
  - Recognition of such errors, particularly those that recur for particular students, can lead to processes of self checking and success or a recognition of understanding that has been gained.

In summary then, the behaviours and strategies of adults do not always mirror their understanding, but are a function of context and belief. The next chapter begins to explore the interaction of these beliefs with learning.

## **Chapter Seven - New and old knowledge**

### **7.1 Introduction**

Emerging from the work so far, from both the literature and from the findings considered in Chapters 5 and 6, have come the ideas that adults have a 'reservoir' (Knowles, 1980) of previous experiences containing a mix of correct and flawed mathematical understandings (Duffin and Simpson, 2000; Karsenty, 2002; Schoenfeld, 1992) and that this collection of knowledge and beliefs forms the base which people use to 'do mathematics'.

The reception of new knowledge is also dependant on this base (Black, 1999), no matter how imperfect this 'old' knowledge and understanding might be. Therefore, the third research question considering the interactions of old and new knowledge could not be considered in isolation from the previous two research questions. Indeed, in some ways, this third question could be seen as the culmination of this research since it considers the learning itself.

The first few sections of this chapter consider some of the remaining ideas to emerge from the transcripts using the method of analysis previously described in Section 4.3. These included explicit references to interactions and excerpts relating to school, teachers, work and memory and sections of dialogue which showed a teaching intervention followed by a student reaction, a learning event. The later sections of this chapter unite these new findings with ideas from previous chapters to generate a series of conclusions relevant to Research Question 3.

### **7.2 Student Awareness of conflicts**

During the interview, students were asked whether they were aware of any occasions when the mathematics of Foundation had appeared to conflict with the

mathematics they remembered from school. Long before the start of this research, I had introduced the concept of BODMAS to a group of 'taster day' adults. The high level of conflict voiced by one particularly articulate student had caused me to reconsider the way I introduced this idea. In asking interviewees this question, I was perhaps expecting examples of occasions such as these or smaller scale times of difficulty matching experiences.

Interviewees indicated that conflicts had not happened, or people had not been explicitly aware of their occurrence, or they were no longer remembered. Since the BODMAS incident, my teaching activities on this subject had been redesigned specifically to avoid such a dramatic conflict again and perhaps it is only events of this type that might be remembered. Alternatively, using Duffin and Simpson's (2000) interpretation, it was possible that students avoided conflict by identifying new knowledge as alien, i.e. not connected with what had been learnt before.

The response from Ann seemed to summarise the general feeling:

*'There were some little different things but they were marginal ... I remember thinking at the time, oh that's a little bit different, but now I couldn't tell you what it was so it obviously wasn't anything too major.'*

Cognitive conflict (Piaget and Inhelder, 1958, Skemp, 1979) was not a familiar concept for interviewees (except perhaps the psychology student), since no-one mentioned issues like multiplication makes bigger despite having just looked at it. Although students were not explicitly aware of conflicts between old and new knowledge, discussions during other parts of the interview provided some valuable information about interactions between them.

Christine saw the Foundation mathematics as a simple continuation of her school mathematics after a break during which she had forgotten it. New knowledge was



assumed to be the same as old knowledge and slotted back into gaps.

*'It was a lot of refreshing of what I'd forgotten from when I was at school cause I noticed as we were going through the lessons I was like oh yeah I remember that now. But because I hadn't used it every day or in a long time it had just gone away from me.'*

The mathematics itself was seen to be the same, perhaps reflecting a subconscious belief in mathematics as one orthogonal truth. As Ann said of the Numerical Skills class:

*'Why didn't they just call it maths?'*

There was sometimes a recognition that something had changed to enable them to understand better, but these changes were usually attributed to people, either the increased maturity and changed attitude of the student or the sympathetic or different teaching style of the teacher, not to the mathematics.

Like Christine, Ann appeared to indicate much was the same:

*'a lot of the stuff that we did I sort of remembered from school as we went through. I could remember and I was thinking 'oh yeah we did that but it was slightly different' and things come back to you a little bit'*

However, her description resonated a little with reconstruction (Duffin and Simpson, 2000; Karsenty, 2002) rather than remembering, since she recognised it wasn't quite an exact match.

Brenda clearly linked Foundation mathematics with the work that she had done at school but considered it more relevant because it was tailored towards the directions she wanted to go (Knowles, 1980) and interacted with other subjects:

*'At school, you wonder why you're doing it sometimes. It became apparent in chemistry when I found the chemistry a lot easier than I would*

*have at school erm I think at school the calculations I couldn't get my head around whereas now I think this is maths, erm you know number of moles and balancing things, balancing equations and all that.'*

For her, there appeared to be no elements of conflict. Foundation mathematics gave her the opportunity to go through school mathematics again, filling in the gaps and offering an opportunity to revisit the areas she felt she hadn't quite had time to grasp first time round. For some topics she seemed to have valued starting again rather than filling in gaps. She compared algebra at school:

*'At school we didn't seem to start at the beginning. I didn't miss any time at school so we seemed to just muddle through it.'*

with algebra on Foundation:

*'so we did it much more methodically .... we started with very simple equations and moving on .... so in that sense there was a more logical approach to it and it was methodical and step by step.'*

Interestingly, the series of algebra worksheets used on Foundation were cited previously in Section 2.1 as an example of the use of a Gagné instructional approach (Gagné, 1968, 1969, 1985) where sets of sub-skills were developed and consolidated before moving onto the next. Her response seemed to support the suggestion that sometimes students valued drill and practice (Safford, 2000).

Ann also found that Foundation and school mathematics seemed to fit together:

*'No, well I'll try and switch between the two, whatever suits me for that particular problem, sometimes I'm not even aware that it's the way I did it at school, it's the way I did it here I just sort of try and pluck it out, which I think is the best or the easiest way for me to solve the problem.'*

Ann was recognising that this was not new knowledge, or even a new approach,

but that she now had different skills:

*'I think generally most things I recognised, though this time around they seemed a lot clearer and I don't know if that's age or confidence on my part but there were a lot of things that were gone through the same way as when we were at school but they were more accessible.'*

In some ways, Ann's statement supports the earlier suggestion that some of her 'memories' were in fact a mix of memory and reconstruction (Duffin and Simpson, 2000; Karsenty, 2002). Revisiting consolidated the memories and replaced some of the flawed reconstructions with stronger, more accessible, alternatives.

Brenda appeared to show again her perception of lack of conflict with school mathematics when she argued that her difficulty with simultaneous equations was because she had not met them before:

*'Oh the one thing I didn't do at school was simultaneous equations, we didn't do those at all so I think that's why I struggled a bit cause erm it was the first time I'd encountered them.'*

When Brenda's statement was considered in the light of her whole interview, this statement could be interpreted in different ways. She had a Grade C at GCSE so it was possible that she had, indeed, spent little time studying simultaneous equations in school. However, she had previously identified that the teaching of algebra in school had left her muddled and she felt that she had benefitted from the more Gagné (1968, 1969, 1985) style approach to basic equation learning used on the Foundation Programme. This did not suggest that she truly believed a school experience in algebra would have helped. I suggest that another interpretation was her need to rationalise her recognition that she found this difficult with her increased levels of self-efficacy (Bandura, 1994) that suggested

she ought to be able to do it. The blame had been apportioned to something that happened before she arrived and over which she had no control.

There seemed to be a number of occasions where the maths gained from real life was invisible (Coben, 2000). Students seemed unwilling to attribute learning to out of school activities and when these skills became visible they were attributed to other learning experiences, as Ann's comment showed when considering a 'real life' task of reading from a census table:

*'I don't have a definite memory of doing that in school, but I didn't feel uncomfortable with it so I thought I must have done.'*

Similarly, Brenda stated that she felt better about mathematics now because she had a clearer or more logical approach than she had in the past but failed to recognise this as part of the numeracy skills set.

Sadly, Elaine was unable to contribute much to this area of discussion (see Appendix 5 for information about Elaine). She said that she recognised some of it:

*'I mean algebra and the angles, triangles and stuff we did at school but a lot of it as I said got thrown a book,(reference to being told to sit in a corner) I probably did do some of it but It was a foreign language.'*

It appeared interaction between new and old ideas might be best studied when those interactions were taking place.

### **7.3 Learning Events –The stories**

Hidden within the interviews were a number of teaching/learning interactions, particularly involving use of different methods for percentage calculations and discussion on MMBDMS. When these interactions were put together for each student, learning stories began to evolve which I tried to interpret using Confrey's (1991) ideas that people proceeded in ways that were internally consistent.

In the percentage learning events, the aims of the teaching were slightly different in each case and no two students started from the same place. However, people broadly fell into two groups, those that preferred formal methods and those that preferred informal. I wanted to move those using building methods (Lamon, 1994) based on finding 10% and 5% towards accepting the alternative semi-formal method of finding 1% and multiplying. This was not intended to replace the original method but to provide an alternative that might be more efficient in certain circumstances. This might also provide a useful starting point for developing or justifying more formal rules by induction in the future.

For those using formal rules, I wanted to revisit informal processes so that these might be seen as a tool for checking answers or indeed for calculating them. (In questionnaires, some students had written down the correct calculation but been unable to find the answer itself without a calculator.) I hoped that revisiting problems from a more 'real life' basis might help people who tried to use the same calculation for every type of question. As with those students preferring informal methods, I thought that the idea of 1% and multiply had the potential for extension to more formal rules by induction, but crucially this linking between informal and formal methods might provide an opportunity for people to understand why different types of percentage problems required different calculations.

I wanted to use the process of reinforcing and extending discussed in Section 2.2 in response to ideas on self-efficacy (Bandura, 1994), domain (Swan, 2001), the Zone of Proximal Development (Vygotsky, 1978), scaffolding (Wood *et al.*, 1976) and the assumption that people proceed in 'personally reasonable and productive ways' (Confrey, 1991, p.111). Since each story occupies some pages of narrative, the learning events are first summarised in Table 7.1

Table 7.1: Overview of Learning Events			
	Issue	Short term aims in interview	Outcome
<b>Anne</b>	Used same formal rule (divide one number into the other and x by 100) regardless of question type Except reduction question (5% of £20) where she found 10% and halved	Develop idea of find 1% and multiply Extend to other questions Show that this sometimes enabled an answer to be found without a calculator Begin to identify difference in type	Swapped. Now tried to use new rule (find 1% and multiply) regardless of question type
<b>Brenda</b>	Used correct formal rules but was unable to actually calculate answer. No acknowledgement of informal methods	Revisit/Develop some informal methods Show sometimes more efficient for generating an actual answer without a calculator. Provide an alternative and/or method of checking	Wanted to stay with the method that worked.
<b>Christine</b>	Sometimes successfully using building	Extend to find 1% and multiply Show might be more efficient than building for some questions Embed in toolkit ready for extending to more formal methods in the future	Some movement but wanting to retain teacher's methods
<b>Diane</b>	Always successful. Usually building or finding 1% and multiplying	No clear aims with regard to percentage	
<b>Elaine</b>	Never successful	No clear aims with regard to percentage	Some way to go

### Ann's story

During the initial questionnaire, Ann had confidently and competently found the correct answers for all the percentage questions, whatever the type, although she had rarely shown working. However, whatever her methods might have been, she had been unable to extend them to the questions in Part B which involved more complex numbers, despite being given access to a calculator for this part.

In the interview, Ann found 5% of £20 by first finding 10% and halving it and indicated that she had done so because:

*'they were round numbers they were like a 20 and a 5 so that was quite easy to do in your head.'*

She also appeared to have been influenced by number when she attempted to find 24 out of 800 as a percentage and spotted that 8 went into both numbers although she then interchanged 8 with 3.

*'what I would probably do on this one, because they can both be divided by 8, I would know that would be 8 out of 100.'*

For all the other percentage questions when she:

*'couldn't see immediately how it was going to, how 6 would go into 25?'*

she elected to use her 'formula' which could be summarised as:  $\frac{a}{b} \times 100$

There was some evidence from her statements that she felt this to be a more advanced method. When she decided to use building for 5% of £20 rather than her formula method, she stated that she was reverting:

*'to do that I would probably revert back to the other way of doing it, find 10% which is £2 then half it to find the 5% which is £1 take the £1 from the £20 and come up with £19.'*

The word 'revert' seemed to indicate her belief that her old method may have been

more illicit (Duffin and Simpson, 2000). She perceived her formula method to be more legitimate or acceptable in academic problem solving, perhaps indicating her recognition of the need to 'meet the expectations' (Leron and Hazzan, 1997) of her new 'community of practice' (Lave and Wenger, 1999). This idea was reinforced by her statement about the interaction of context with task selection (Lave, 1988 ; Maier, 1991; Roth, 1999) with her earlier mention of an exam (see Section 6.3).

She used the same formula for all percentage questions, when it was a appropriate and when it was not, but on no occasion did she actually calculate a numerical answer because as she put it:

*'I can't do that in my head unfortunately.'*

When it was suggested that she might use a similar method for finding 6% of 250 to the one she had used to find 5% of £20, she noted:

*'I suppose another way to do it would be to find 1% of 250 then multiply that by 6? ... but that would give me 1% would be 2.5 times 6 which is 15.'*

When it was later suggested that she could find the answer to 43% of 800 by the same technique, she agreed that she could:

*'find 1% of 800 .... and multiply it by 43.'*

but rejected this as an option because:

*'I couldn't multiply 8 by 43 in my head.'*

This was an interesting response, given that she had not been able to work out her answer using her formula in her head either. Was the difference again in the legitimacy of one method over another?

During the interview I tried to highlight why her formula was incorrect for one question and not another. For example, I compared the calculation she had done using her formula ( $\frac{6}{120} \times 100$ ) with the calculation via 1% ( $\frac{6}{100} \times 250$ ). I also



emphasised that questions were different. Despite this, Ann abandoned her formula for the last question and tried to calculate 1%:

*'so on that one you need to find what percentage 24 is of 300 Erm 24%, 300 (long pause) I would revert back to, again you could do the find out what is 1% and then divide 24.'*

Unfortunately, of course, this question required her to convert a proportion into a percentage. Her formula would have been an appropriate method for answering this question, trying to find 1% was incorrect:

Interviewer: *Careful what does the question actually want?*

Ann: *Oh it wants the percentage, what percentage is 24 of 300 (long pause) I can't remember.*

Two particular ideas emerged strongly from Ann's story. First was her belief that some methods were more appropriate in an academic setting (supporting ideas from Duffin and Simpson, 2000; Karsenty, 2002; Lave, 1988; Lave and Wenger, 1999; Maier, 1991; Roth, 1999). Secondly, that once she has found her formula, she would stick with it, although it was not clear whether this was a variation of belief in 'the one right method' (Coben, 2000; Schoenfeld, 1992) or staying with something that appeared to give a satisfactory answer rather than searching for something deeper (Lithner, 2008).

### Brenda's story

Brenda had completed all her percentage questions in the initial questionnaire using formal calculations. At no point did she use any informal methods. She varied the calculations that she did, and did not always select an appropriate one (5 out of 7 were correct). In her follow-up questionnaire Brenda selected an appropriate method for every question.

In the interview Brenda again selected to use formal calculations and produced an appropriate method for every calculation using percentage although, like Ann, there were occasions when she said:

*'But I couldn't work it out in my head.'*

I tried to use scaffolding (Wood *et al.*, 1976) to lead her through an alternative method for finding 5% of £20:

Interviewer: *If I asked you £20, what's 10%?*

Brenda: (pause) 2

Interviewer: *What's 5%?*

Brenda: (Mumbling) *no I couldn't do that.*

Interviewer: *So 10% was 2?*

Brenda: *Ok then 1.*

Interviewer: *1 so you found 5% is £1 without using a calculator.*

Brenda: *See I wouldn't have realised I would just look at the numbers and wouldn't have thought about 10%.*

Interviewer: *... a lot of people started off like that and then developed that method you were using afterwards, but you can't remember ever doing it like that?*

Brenda *No to be honest I can't even remember doing percentages since December a few years ago, when I sat down with a maths book and went over a few things so I just did it that way, if I had a number like 20 over 100 I just cancelled down straight away with the noughts and work it out that way.*

Brenda later agreed that she was aware that other methods existed but she preferred her own. She identified that other methods might be successful and

better for other people:

*'They did about 3 different ways of doing them and it was interesting to see, because I have 1 way of doing things and it's the way I've been taught but people in the class said 'oh I've done it this way' so we ended up doing 3 different ways of doing them but I still felt comfortable with the one I knew and try to get to grips, I remember ... had quite a different way of doing it but I couldn't get to grips with that, and I think it's because it had been instilled in me so much that I just felt more comfortable so I'd just stick with that but it's interesting to see a few different ways, that was one that stuck in my mind.'*

I asked her:

*'Did you shut your mind when the other methods came up or did you listen and reject it?'*

She responded:

*'I listened to them and I think I tried, especially and I can't remember how she did it but I just couldn't do it so I thought, I'm probably better off sticking with what I know if it works, so as a result I can't really remember what the other method was erm but it was interesting to see such a different method.'*

The reference here to finding a method that 'works' suggested an interpretation of her behaviour as deciding not to search further for alternative methods because she had found a satisfactory one (Lithner, 2008), but her reference to 'trying' others refutes this. Her description resonates more with Duffin and Simpson's (2000) designation of 'alien', where she had elected to ignore (evidenced by the fact she could not remember what the new methods were) or avoid by simply

sticking to her old method. At the same time she was perhaps aware that the new methods was not really alien and her behaviour might be at fault, hence the need to justify her behaviour: 'I tried' (prefaced by 'I think' suggesting she's not really sure she did), 'it was interesting to see' (mentioned twice perhaps trying to suggest that she really didn't try to avoid them) and her final justification, the teacher did it too:

*'I think that's what she had been taught at school that worked for her so she stuck with it just like I stuck with mine.'*

and later:

*'so if we were doing percentages, she would use her method that she was more comfortable with, I think it was just to find a method that we could all do and we each could find a method that worked for us rather than just one way of thinking.'*

At the end of the interview Brenda summarised her experience with maths on the Foundation Programme when she said:

*'and I think, 'oh yes I'll try that' and now my approach is much better and when I encounter maths in the future, which I will do, hopefully I'll approach it and know which rules to work with.'*

Within one statement, she appeared to be both suggesting that she was willing to try new ideas and keen to stick to rules.

### Christine's Story

Christine appeared to have had a strong school experience which had clearly influenced her approach. She mentioned school and teacher far more often than the other four interviewees. She talked of tables and testing and mentioned ideas being instilled. Her teacher appeared to function not only as a 'significant other'

(Coben, 2000; Karsenty, 2004; Safford, 2000) in that she had been heavily influential in Christine's ideas of mathematics, but also an invisible 'authoritative other' (Leron and Hazzan, 1997) in that Christine sometimes appeared to be matching her actions to her previous teacher's expectations within the interview. For her, success was still sometimes seen as ratification by the teacher (Schoenfeld, 1992).

In her initial questionnaire, Christine had usually elected to build. When she had needed to write a proportion as a percentage she had sometimes used a form of building and matching (Section 5.3). She had managed the more complex questions in part B and it was possible that she had used a more formal calculation for these. In her follow-up questionnaire, she had successfully found 5% of £20 which she had not done before but was still using mainly building.

In the interview she was sometimes very clear about the calculation she was doing and where it had come from:

*'6% of 250, when we did that at school, we used to go it's closer to 10% so you do 10% which is 25. You need half of that for the 5% which is 12.5 and then you have to find 1% of the 12.5 to give you 6 % ... We did it in sections.'*

When asked to find a proportion as a percentage she recognised that this was a different kind of question and that her build and match method had had limitations:

*'I'll have to think about that, (mumbles for 8 secs) if I remember right we used to do it roughly the same way but it didn't always work when I was younger, I couldn't, I couldn't do that percentage and a number, I could do it the other way but I always had a lot of trouble doing it.'*

When I explained a possible alternative, to write the proportion as a fraction, her response was:

*'Oh yeah I never thought of that.'*

and slightly later:

*'I didn't know we had to reverse it, just I got really confused.'*

As we moved through the interview, I attempted to see whether she would accept a slightly more formal method for her calculations, that of find 1% and multiply.

Although she was willing to be led through such calculations and accepted that the answer was correct, she rejected it as an appropriate method for her because she was unwilling to change from the way she had been taught:

*'Yeah I could have done it like you said but I'm one of these, I can't, I like sticking with things the way I was taught.'*

In some ways Christine's behaviour was similar to Brenda's. She reverted to her familiar method. However, she did not appear to be avoiding or ignoring the other method, indeed, she showed that she could have used it. She appeared to have made a value judgement, if not that there was one right method, certainly that there was one best method and this is the one her teacher had used.

She was initially unwilling to accept that building methods might be more difficult for more complex numbers, partly because she was very competent in using her method. As with Ann and Brenda previously, the apparently satisfactory results, provided little motivation to search for alternative methods (Lithner, 2008):

Interviewer: *You see 6% is quite nice cause you've got your 10% and 5% and your 1%, but suppose it was 17%.*

Christine: *I'd do 10, 5 and then 2 sets of 1.*

I persevered. When the same question 6% of 250 was revisited in her

questionnaire, she had actually made a numerical error and stated 5% as 12 rather than 12.5. This gave me another opportunity:

Interviewer: *There's your 5%, you actually went a bit wrong because of a rounding error.*

Christine: *Ah right, yeah 5% is 12.5, I can see that now.*

Interviewer: *And one of the reasons why I would recommend going directly to 1% is if you went from 250 and you wanted 1% you could see it was 2.5, and that would have avoided those calculations where there a two chances to go wrong, if you have got one calculation there is only one chance to go wrong.*

Christine: *Right I'll have to remember that.*

As the following extract shows, there appeared to be a gradual shift within the interview from always finding 5% first to sometimes finding 1% first. What was particularly interesting was the subtle change in her view of what the teacher might actually have said. If her final statement in this extract is compared with the statement reported earlier in this writing, her memory appears to have been reconstructed (Draaisma, 2004):

Interviewer: *So there you did 5% of the total is 12.5, how did you get the 1%, did you get it from the 5%, or did you get the 1% from the 10?*

Christine: *I think that's how I did it, well I got the 1%, cause they are the easiest numbers to work with, the 10 and the 1 so I'd got the 10%, then because I needed so many I had to have the 5% as well.*

Interviewer: *I think, or what it looks like to me is that the best method might be to find out 10% to find out the 1% and then you do what ever the % time the value of 1%.*

Christine: *That's what our teacher used to do, she used to do 10% is easy*  
*cause you just move the decimal, and then you go down to the 1%*  
*which is moving the decimal point again and then if you need*  
*anything in between half of 10 is 5 and then you can still work out*  
*any percentage you need from that.*

It appeared that this notion of sticking with the way she was taught was crucial in her decisions about learning. For this reason, I used a part of the interview to focus on exploring and encouraging flexibility.

Christine was aware that different people found different approaches helpful. She discussed the differences between her two children and reflected on her own decisions to change method:

*'I have had to break things down for her and when I get a bit confused if I just, you know some days you just get confused over silly little things, I go, if it doesn't work the first time with the way I taught myself, I do it the way I taught my daughter which is like basic step by step, you go all the way down if possible.'*

She also referred to being taught different methods:

*'I was taught several different methods and some of them suit some, but I don't like that, I tend to just look and go I don't like that one can I have some .... I like them.'*

The following extract shows part of an attempt to encourage flexibility and openness to new methods attempting again to use the principles of positive reinforcement, extension and listening with, perhaps, partial success:

Interviewer: *you have to be very careful is that the method that works for you*  
*(positive reinforcement (Bandura, 1994)) is a true method (reminder*



of MMBDMS) *you say well it always worked for me, and then you try and stick it somewhere else and you're in trouble so it's identifying whether the method you've got is true in all areas (extension of domain (Swan, 2001)) and then the other one is whether the method you have got is the most efficient for all areas. (talk about 2394 x 478 by repeated adding of blocks or multiplication) There is nothing wrong with that first method, just it will take you a long time and you might need an awful lot of space so there comes a point where it is no longer the most efficient way of doing it. ... you move on taking from one idea that you have got and moving on a stage further. So from that 5% we've developed the one percent and we are saying, perhaps it's easier to go straight to the 1%.*

(Note: positive reinforcement again, with the words of 'perhaps' and 'easier' rather than 'definitely better'.)

Christine: *I would never have thought about going straight to the 1% because it's like a process with the decimal point, just moving the decimal point, so if I go to the 10% I have to move it once and then if I go to the 1% I have to move it again, I've never even sat and thought, I'll just move the decimal point twice. But that would be quicker.*

(further discussion)

Christine: *Alright, I'll have to try and remember the 1%*

(further discussion)

Interviewer: *Right so if a rule works for you, you can't assume*

Christine: *it's gonna work for someone else*

It appeared from this interview that it was possible to gradually change Christine's

ideas through scaffolding (Wood *et al.*, 1976) but that she was being held back by her belief that her teacher had given her a good method that worked. She recognised her own limitations with mathematics, in relation to her proposed career as a primary teacher:

*'I know it's going to be difficult and when you have to teach something you struggle with to somebody else, I know you get difficulties.'*

but at no point recognised that her teacher's methods might have been limited too. This raised an interesting question. Should the focus of Christine's future teaching be on the maths, or on the belief?

One final extract from Christine's interview provided an additional question. Was it only for some topics that she believed her school teacher had given her the best method? In referring to differences between Foundation and school she stated:

*'There were a couple of things where I've been taught at school a different way ... you had a couple of things where you explained it so much better and I understood it a lot more than when I was at school ... I sort of adopted some of the things I was taught by you, cause I thought well that's so much easier to understand ... I just thought 'that is so much easier than the way I have been trying to do it in my head.'*

Unfortunately, she found it difficult to recall exactly what those 'things' were.

### Diane's Story

Diane used similar methods to Christine in her initial questionnaire but with more success. She interchanged finding 5% and building with finding 1% and multiplying as appropriate. Like Christine, she used a method of build and match (Figure 5.13) to find proportion as a percentage. In the follow-up questionnaire she used the same methods with equal success.

Diane believed that she was better at percentages because of her work in a shop.

*'The percentages aren't too bad for me because I used to work in a shop so I kind of know that when you take 10% off something it should be this.'*

However, her response later when she met a series of non-shop based percentage questions:

*'O, I hate these ones!'*

suggested that this experience may not have been as helpful as she had supposed.

She was also more open to reflection and new ideas as shown in the following excerpt discussing whether division order made a difference.

Diane: *Would it give you a different answer if it was  $0.4 \div 8$  would that give a different answer altogether?*

Interviewer: *Yeah, want to try it?*

Diane: *Yeah, Just out of curiosity, ... ok so the  $8 \div 0.4$  is 20, that's right but then if you do  $0.4 \div 8$  it's 0.05, Whether I mentally change it round automatically and think well it must be smaller but that's really surprising.*

During the interview, Diane continued to use building or finding 1% but had identified that her methods were perhaps no longer the norm for other students.

She stated:

*'I work out percentages so strange, don't I. '*

She indicated that she had also developed some more formal calculations as the following excerpt shows, probably triggered because she didn't realise that she could use the pen and paper provided:

Diane: *If 4 out of 100 avenger cars .... am I allowed to use a calculator?*

### Elaine's story

Elaine had little idea about percentages either in the questionnaires or in her interview. Her first reaction on meeting them in the interview summed this up:

*'Oh crumbs percentages lovely, I don't remember what you do with them, do you multiply them by 100? Oh gosh.'*

Her reaction later after a somewhat high speed explanation indicated my tuition had not been very successful:

*'I knew somewhere you had to times it by 100 with a percentage.'*

Perhaps her own assessment at the end of the interview summarised my own conclusion:

*'I still can't get my head around percentages, I shall have to sit and work on them.'*

Taken together, these five stories generated some insights and some dilemmas.

The most obvious conclusion was, I believe, the most important and a fundamental part of this study. All five students were totally different. Two preferred to use less formal methods of calculation but one of those appeared fairly successful and the other not, possibly linked to different views on changing methods. Two preferred to use formal calculations and again one was successfully selecting appropriate methods that worked whilst the other wanted to use the same method for everything. The fifth student did not really have any methods that worked.

Before drawing more conclusions for future practice from these learning/teaching interactions, there needed to be some clarification about purpose or desirable outcome of such teaching. An expert mathematician is able to link and switch between all existing methods selecting the most appropriate based on efficiency or

availability of calculation tools. The students considered here, progressing from the Foundation Programme, needed to be competent and confident in performing calculations involving percentage and with calculators available. These two outcomes were not mutually exclusive. However, they might not be mutually achievable for the majority of the Foundation students, given the limited time and resource available. In line with other decisions made throughout this study, the latter outcome, being competent and confident took priority.

From this viewpoint, Brenda's Foundation experience had been successful. She had begun the programme with a number of formal methods but some confusion about which method to use and when. She had not been explicitly aware of this confusion but during the course, possibly when other people's methods were being considered, she managed to consolidate her own knowledge and become fully competent. Although Ann's experience had been less successful, there would have been little direct value for either her or Brenda to have been re-introduced to less formal methods (or go back to the beginning) only for the tutor to prove by induction that the formal methods they already used could be justified. It might have been more appropriate to work with Ann's belief of 'the one right method' and to provide a 'formula' for each type of question. Effort could then have been expended on developing the skills of problem recognition not solution.

For Diane, competent in less formal methods and already interchanging between them, a linking of methods might have been valuable. However, a theoretical justification for the movement from informal and semi-formal methods to more formal ones on a blackboard would have had little value for Christine. She needed to travel on a journey closely linked to her own Zone of Proximal Development (Vygotsky, 1978) to gradually move her away from reliance on her teacher's

methods. Perhaps Christine might have benefitted from starting in a different place, from being given a formal set of calculations which were quite clearly different from those of her teacher despite the potential risk of disempowerment through abandoning strongly held views.

## **7.4 Helpful Ideas from Previous Chapters**

It was previously identified that the learning adults might bring to their foundation classes was not restricted to specific maths areas but included certain beliefs and behaviours. Adults tended to take control of their own learning and it was suggested that involvement in a self-efficacy/success loop (Bandura, 1994) meant they might be more likely to continue with a task than give up. This willingness to complete a task might mean a greater likelihood that adults would achieve equilibration if new ideas appeared to conflict. My research showed both that the 'Multiplication makes bigger' overgeneralisation was common amongst foundation students and that there was a clear link between the success in some questions and this belief (see Section 5.4). Within the five interviews students appeared willing to adjust their schema to accommodate these new ideas. However, it was also noted that sometimes students classed new ideas as 'alien' (Duffin and Simpson, 2000) and this avoided the need to deal with the conflict at all (see learning stories). There are many examples where the conversation approach has proved successful, particularly amongst mature adults for example, Buxton (1981). Students did adopt new methods but often dropped their original one. One of the most obvious examples of this was the change from the more spontaneous build up methods for finding 6%.(find 5% and 1% and add) to variations on  $6/100 \times$  . The old method was not lost but relegated to the shops. The new method was

seen to be more appropriate for the new environment. The notion that both could be used or one to check the other seemed unknown.

It was noted in Chapter 6 that adults appeared willing to check for reasonableness and to continue until this was achieved (Benn, 1997), a potentially useful skill in developing new learning. Adults might also bring skills of estimating and recognition of sensible answers which could be both a help and a hindrance to new learning, particularly if the frequent use of such methods had reduced the need to keep other skills sharp. Some students for example, were found to perform a range of alternative procedures and then select the sensible answer, a method which could not be extrapolated to situations where sensible answers could not be recognised. Further, this method was particularly unreliable if the correct and incorrect answers were very similar.

There has been some discussion about the use of intuitive methods and particularly of being misled by semantics, number and 'familiar faces in a crowd', (Leron and Hazzan, 1997). This is not unique to adults (see for example Hart, 1981b). However, there are occasions when mathematics itself is counterintuitive. Certain fixed beliefs can have a detrimental effect on the acquisition of new learning. The belief that the purpose of mathematics teaching is to find the answer, not the route to get there, leads to the label of cheating attributed to calculator use as challenged in one of the interviews, yet my research has suggested that the use of calculators can provide cognitive economy (Cowan, 2003) and allow greater opportunity to identify process or keep track. The belief that it is cheating to check answers can lead to the consolidation of incorrect processing and development of incorrect associations (see Cowan, 2003). Indeed previous incorrect associations can already interfere in the acceptance of new

ideas. The repeated discovery of the incorrect recall of certain key facts by some adults confirmed that adults have acquired a number of false associations. As I have discussed earlier, the effect of such errors within new learning is to divert attention from issues of understanding and lead to potential disempowerment.

## **7.5 Conclusions**

It appeared that, in general, students were not explicitly aware of conflicts between old and new knowledge. This might sometimes have been due to elements of reconstructing of previous knowledge. Further the belief that there was one right answer, one right method and in effect 'one mathematics' might have caused people to unconsciously adjust their observations to fit into this framework. Thus, they were not predisposed to look for cognitive conflict and, since they did not expect to find it, classified such interactions in alternative ways. Their early responses often fitted with assimilation (Piaget and Inhelder, 1958).

Skemp's (1979) suggestion that some heavily embedded schema might be resistant to change and Swan's (2006) suggestion that people might go to some lengths to keep logically incompatible schema apart was supported by some of the behaviours within the interview, particularly the resistance to considering new methods when 'one that worked' had already been found. This fitted with Duffin and Simpson's (2000) ideas of ignoring or avoiding 'alien' learning but might also have fitted with Lithner's (2008) suggestion that if an answer appears satisfactory there may be little motivation to look for alternatives. This reluctance to change, to accept that new methods may be more appropriate or more efficient in new circumstances might be preventing future development. There was a need to encourage selection of most appropriate or most efficient tools for purpose. The key appeared to be to make such notions explicit, to help identify when previous



methods would be helpful and when and why they would not, in effect, to move teaching towards conversations such as those held in the interview.

The five learning events extracted from the transcripts showed a process of reinforcement (Bandura, 1994) and extension through attempts to scaffold (Wood *et al.*, 1976) students from where they were to where they could be (Zone of Proximal Development, Vygotsky, 1978). It was evident that learning was building on a base of both understanding and belief. It was not just the mathematical knowledge that needed to be scaffolded but also the beliefs that accompanied it.

The previous chapters showed that successful completion of a problem did not necessarily mean that a student had the same understanding as anyone else. Students had used a wealth of methods to find solutions including intuitive methods, trial and error and those suggested by semantics and number, with some of these methods not explicitly known to the students themselves. The issue was potentially further confused by the impact of cognitive errors. Thus, a student might be in a very different place on a very different learning journey from that envisaged by the tutor.

This mismatch, and perhaps just as importantly, this lack of awareness of the possibility of mismatch, was for me a major discovery. The notion that a student might not be in the same place as I believe them to be, offers both a possible explanation and a potential means of improvement for those occasions when learning appears to have reached a total impasse.

The learning stories just described showed how individual conversations helped to identify the starting point for each learner both in mathematics understanding and belief and increased the potential for successful scaffolding.

## 7.6 Key Findings for Research Question Three

The third research question asked:

**How does old learning and learning from other situations interact with new learning?**

The following findings appear to emerge from the conclusions just given.

- 1) Students appeared to have predominantly adopted the fill in the gaps framework for classifying changes in their learning and were not predisposed to search for or identify cognitive conflict.
  - They had forgotten something and new teaching reminded them so they slotted it back into the gap. There was no recognition that it might be different although there might have been occasions when memories were being reconstructed.
- 2) Their belief that all maths was the same sometimes led them to believe that the methods used must be the same and thus improved understanding was attributed to changes in the student or changes to a more sympathetic or understandable teaching style.
- 3) When they remembered having difficulty with a topic in the past, there was evidence that some students were conscious of making the decision to 'forget' their old learning and replace it with the new.
- 4) When they trusted an old method and believed it worked for them, none of the students interviewed said they were willing to replace this method with a new one. None of them consciously identified that one method might be more efficient than another.
  - There appeared to be a strong belief that if a method had worked for them so far it, would also work for them in the future. If a method appeared

- successful there was little motivation to search for alternative methods (Lithner, 2008). Sometimes, alternative methods appeared to be classed as 'alien' (Duffin and Simpson, 2000) and were thus ignored or avoided. In effect, having a method that worked potentially held back new learning.
- Students were sometimes using different methods between questionnaires but this may have represented a belief in change of task and therefore different acceptable outcome.
- 5) In a similar way to the 'one right answer' idea, there appeared to be a 'one right method' idea, or at least the notion that you could only adopt one.
    - There was a choice to be made. A new method could be adopted for a new problem or different environment but you couldn't have two and select the best or use one to enhance the other.
    - It seemed acceptable to gain new methods in a new context because there was a new hole for the new knowledge but it wasn't considered acceptable to have two methods for the same thing.
  - 6) Some knowledge gained in adult life was falsely attributed to school but occasionally some skills were falsely attributed to work.
  - 7) Ideas of reasonableness from adult life were readily transferred to checking foundation maths problems. Some students also used estimating methods from real life to check answers. However, real life estimation methods were not always able to be extended and occasionally held back progress.
  - 8) For successful scaffolding (Wood *et al.*, 1976), the Zone of Proximal Development (Vygotsky, 1978) needs to consider not just the mathematics knowledge but also the beliefs associated with it.

- 9) The apparently successful completion of a problem did not necessarily mean that a student was in the same place and on the same learning journey as that envisaged by a tutor.
- If a tutor is to successfully scaffold from old ideas to new, the assumption above needs to be challenged.
  - If there appear to be difficulties with the development of new ideas, it would be beneficial to explore the true reasoning of the student.

In summary then, adults bring with them a whole range of mathematics memories, reconstructions and beliefs. Future learning will be constructed on this foundation. Successful learning partnerships between student and tutor require identifications of where a student is and where they could be, both in their understanding and belief. This study has highlighted how individualised such understandings and beliefs are and how unexpected some behaviours. Further, it has been suggested that apparently successful problem solving might sometimes cause misleading perceptions of understanding, and lead to failures in scaffolding processes.

The clear message from this work is of the great value to be gained from listening to students and providing opportunities to explore both their actions and the reasons for those actions. Tutors need to recognise that preconceived ideas that a student's learning journey mirrors, or should mirror, the path that they themselves took, need to be challenged. Whilst the next chapter suggests a number of implications for practice, by far the most important one is of the value of 'conversations' of truly attempting to look through the eyes of the adult learner.

## **Chapter Eight – Implications for teaching**

Implications for teaching are best considered in three parts: implications for my own teaching, implications for science teaching involving proportion and implications for teaching in general.

### **Implications for my own teaching**

This research has challenged my assumption that I have always followed an empathic approach towards my students. Consideration of the interactions of multiple behaviours and particularly considering actions from a coping perspective indicated how far I sometimes fall short of this ideal. A greater awareness of why students might select particular courses of action will lead to the redesign of some teaching materials to remove misleading cues and the delivery of more appropriately targeted support when students find difficulty.

As a consequence of this research I am more aware of the sorts of processes that might be in use and recognise that my focus in the past may have been incorrect for some students. A key implication from my reading is the benefit of making student processes explicit to the students themselves (Leron and Hazzan, 2006) and this may be particularly important if a transition to more formalised methods is to be achieved without disempowerment. The notion of using ‘conversations’ as discussed in Section 7.6 appears attractive and there are many successful precedents for such work amongst mature adults (see for example Buxton, 1981).

Some of these changes to my practice can be illustrated through the three examples which follow. The learning events discussed in Section 7.3 highlighted the way in which some students had interpreted a lesson discussion on alternative methods for percentage calculations as an endorsement of their decision to

choose a particular method and an encouragement to stick with it. In contrast, comparison of methods between initial and follow-up questionnaires highlighted a change from informal to calculator based methods, possibly in response to changing perceptions of academic requirements. As a consequence of this research, my teaching of percentage has been restructured. Whilst the session begins as in the past, asking everyone to try and solve a number of 'find A% of B' problems using whatever methods they can remember, the numbers used and contexts of the problems have been changed so that some encourage informal and some more formal methods. Thus, the interactive comparison and discussion session that follows provides opportunities to identify explicitly the value of both informal and calculator methods and to directly challenge the belief in 'one best method'.

Similarly, the teaching session which introduces direct proportion now includes a comparison and discussion of methods selected by different people in the class and attempts to make the influence of number on method choice more visible. On completion of this topic, my marking feedback is influenced by these insights into method change.

Whilst it is not possible to hold deep mathematics 'conversations' as suggested above with all students, a small number who produce particularly confused work in the first few weeks of teaching have been invited for an individual discussion to work through some problems together. This has produced some positive responses.

## Implications for those teaching science using proportion

It has already been noted that the use of proportion is implicit in a wide range of mathematics contexts, but for foundation students, proportion is also used in certain key scientific areas. These often seem to be the same areas that are particularly problematic for a few students, where there is a clear separation between those that 'get it' immediately, those that take a little longer and those that have great difficulty. The areas include genetics calculations for those going onto Biology or Human Science and dilution/concentration problems and 'mole' calculations for those going on onto Medicine and Biomedical Sciences.

The issue does not appear to be one of recognition. The link between these problems and proportion was certainly explicit for Brenda when discussing the snake problems:

Interviewer: *Can you see any similarity to chemistry?*

Brenda: *Because you have to work out relative amounts, yeah number of moles and stuff.*

Tutors both in the application subject itself and within Numerical Skills classes tend to try and invoke 'logical' methods, building up from what is known. Use of variations of the unitary method, (though not called that by the science practitioners), is common, as is the replacement of more complex numbers with easier ones to 'see what you are doing and replicate it'. Emphasis is placed on the starting point, extracting the numerical information from the question and writing it in such a way that it can be built on and in finding the link. This research suggests reasons why, for some students, these approaches might not be helpful.

If the selection of the calculation itself is in the context, semantics or order in which

numbers appear, the method used in one problem simply cannot be used in another. Nor is building and adding the same as multiplying. If 'easier' questions are answered by instinctive methods suggested by number patterns then it is not possible to 'replicate the process' with more complex numbers, particularly if a student is not explicitly aware of what that process was. Further, evidence from my research and reading suggests that some students deliberately set out to *change* the process when numbers become more complex. The choice of method is dependent on the complexity not on the similarity of problem type. Finally, if people are using trial and error type methods and searching for reasonable answer, this process cannot be replicated in scientific calculations if there is no understanding of what a reasonable result is.

I must emphasise that I am not saying that the logical approach does not work. For many students it apparently does with some success, but my findings might reduce some of the frustration caused when people just don't seem to get the 'logical' step. To them the step does not make sense in the light of their ways of understanding.

Mole calculations which use the direct proportionality:

$$\text{mass of substance} = \text{number of moles} \times \text{molecular mass}$$

lend themselves to solving by the use of 'triangle algorithms'. These rewrite the

proportionality as a triangle  $\frac{\text{Mass}}{\text{no.moles} \times M_R}$  and this is popular and

successful with students. However, this is sometimes seen as cheating by teachers, as a last resort quick fix, with a feeling that students using this haven't really understood the underlying concepts and may not be able to deal with every problem. In the light of my research, the emphasis placed on the importance of



'understanding the calculation' appears, at least in the case of some students, to be based on false assumptions.

The success of international students in using  $\frac{a}{b} = \frac{c}{d}$  for so many problems makes me wonder whether there would be benefit in formally introducing it to some students. Whilst the work of Hart (1981b) identified that, even if this was taught, it was not in general use and people continue to revert to informal methods when they could, the drawing out of such an algorithm after appropriate 'conversation', (see Section 7.6) might well produce some beneficial results.

Three recommendations emerge from these implications:

1. Teachers need to exercise care in the selection of initial examples. They should avoid using sets of numbers that encourage alternative instinctive methods which are different from those required for extrapolation.
2. If a student is finding difficulty with scientific calculations, it may be beneficial to investigate initial 'understanding' to confirm that apparent 'success' with easier examples has not been achieved using alternative understandings.
3. For some students, there may be advantages in introducing formal algorithms based on  $\frac{a}{b} = \frac{c}{d}$ , rather than attempting to extrapolate from a base that is confused or does not exist.

These recommendations were given as part of a presentation to the Foundation Year Network in July 2012 and followed an interactive session where delegates were given the two proportion questions on stamps (see Figure 6.2) to compute themselves and were then introduced to some of the findings and student responses from this research. This session initiated considerable interest and

discussion prompting one delegate to refer to it as a 'Eureka moment' and supporting my assertion (see Chapter 10) that Foundation tutors may be unaware of the influence of number.

## **Implications for teachers in general**

Whilst the previous section has focused particularly on proportion, my research suggested that, within any mathematical activity, difficulties may be caused if teachers assume that those who obtained a particular answer must have derived it in a particular way which can then be extracted and used in alternative contexts.

The confirmation that the adult reservoir of experience (Knowles, 1980) contains a confused mental map (Coben, 2000) of memories and learning that are potentially flawed was expected, as was the finding that only a part of that retained might be recalled in a particular context. However, the strength of the influence of belief and acquired adult mathematical behaviours (e.g. instinctive calculation, testing for reasonableness and 'acceptable' actions within a university Community of Practice) on the selection of solution methods and strategies, was less anticipated. This highlighted the benefits for future learning of exploring and making these beliefs and behaviours explicit for both student and teacher.

With the practicalities of teaching highly mixed groups and move towards more distance learning, cross university partnerships producing diagnostic and independent learning material are becoming more common. Approaches are becoming standardised and errors categorised from a more judgemental remedial view. This research suggests that shifting the focus towards a more empathic view, focusing on the strengths and asking the question 'Why?' has the potential to improve the learning experience and increase self-efficacy and empowerment.

Two broad recommendations emerge from these implications:

- It is important for teachers to recognise that student behaviours in a particular mathematical activity may be only a snapshot or subset of all their mathematical understandings. 'Failure' does not necessarily indicate lack of understanding but, similarly, 'success' does not indicate that a student has the same understanding as that envisaged by the teacher.
- Exploration of a student's beliefs alongside exploration of their more mathematical understandings has the potential to greatly enhance the learning experience.

In summary, my research suggests the benefits of teachers focusing on what people bring with them, both mathematically and in belief, making these ideas explicit to students themselves and then drawing them forward with the help of these ideas to the next learning experience.

## **Chapter Nine – Future research work**

A number of different insights have emerged from this research, many of which might benefit from further investigation. The four suggestions detailed here are not intended as an exhaustive list but represent my personal, and therefore subjective, view of areas with the greatest potential to generate valuable pedagogical knowledge for those teaching maths or science to adults.

1) This research compared the answers generated by adults on a Mature Student Foundation Programme with the answers generated to similar questions by children in the CSMS study in the late 1970s (Hart, 1981a). Detailed consideration of the adult answers, illuminated by the CSMS insights from Hart and colleagues, alongside consideration of a range of relevant literature (see Chapter 2), provided considerable knowledge about adult problem solving behaviours within a narrow set of problem types. Such behaviours might be expected to be found amongst at least some of the adult learner population elsewhere. Additionally, the differences between the responses of children and adults highlighted the potential differences in the knowledge base (Schoenfeld, 1992) used for the solution of problems. However, this work can only be considered a first stage. Both the selection of participants and the methods of administration were very different for the adults and the children. Particularly, the CSMS testing tool itself contained far more questions on similar topics, potentially increasing reliability for assessment of levels.

An appropriate next stage for this work would be the design a new testing tool on a limited number of mathematics topics. Questions would be developed from those already in use, but semantics and numbers, (i.e. use of primes or groups of multiples) would be systematically changed and there would be an increased

element of repetition. Through collaboration with partners, possibly within my own institution, the final tool could be administered to both adults and children with as much similarity in testing conditions as possible.

2) The interaction of semantics and numbers (Kaput and West, 1994) and particularly the 'non-conservation of operation' (Greer, 1994) appeared surprisingly influential in the selection of mathematical method and ultimate success amongst the Foundation adults. Since some areas of science are particularly reliant on proportional reasoning, and teaching methods may involve extrapolation from perceived 'simple examples' , this is an area of considerable importance that needs further investigation.

3) Through analysis of transcripts it was possible to extract 'learning stories' (see Chapter 7). It was very apparent that the depth of these discussions, together with the subsequent analysis, enabled a much richer understanding of the student's reasoning and, hence, increased the opportunity for guiding change. Pedagogy is about searching for similarities (Simon, 1999), teaching large numbers of students via individual interviews is not an option. However, the use of individual conversations as part of an intervention programme, possibly for those using alternative proportional reasoning, appears to have potential merit. Future research might monitor the implementation and benefits of such a scheme.

4) The questionnaire scripts both for the years discussed, and for subsequent years, contain a vast amount of information, only some of which has been able to be considered within this research. The revisiting of these scripts with fresh pairs of eyes, may have the potential to generate yet more valuable insights or lines of enquiry.

## **Chapter Ten – Originality and Concluding Remarks**

Whilst there is a growing bank of literature on adults **doing** maths in different contexts, particularly to inform school practice, far less is available on adults **learning** mathematics and, particularly, what they bring with them to the process. With the exception of the work by Evans (2000) and Duffin and Simpson (2000) and work on nurse training (Pozzi *et al.*, 1998; Coben *et al.*, 2007), research that has taken place in this area has tended to focus on affective issues and barriers to learning rather than interactions with mathematical understandings. However, this is an important area for concern given the growth in development of return to learn courses.

This study took diagnostic tools developed to consider children's understanding and applied them in the new domain of mature adult understanding.

Interpretations of understanding of children to develop improved learning experiences in schools must, necessarily, be different from interpretations of understanding of adults to develop improved learning in Foundation courses.

Behaviours observed in children can take on new significances when observed in adults. Therefore, this study attempted to build on the wealth of information available about children's mathematical thinking and extend this into the newer area of adult mathematical thinking, whilst at the same time considering alternative influences on behaviour.

The use of such tools for mature students proved a valuable first step in the investigation of adult mathematical understanding and behaviours. Additionally, comparisons between hierarchies of understanding for CSMS children with apparent hierarchies of retention or recall for adults indicated areas worthy of

further investigation. Suggestions for further research developing from this tool were made in Chapter 9 (1).

With the exception of some of the contributions from members of the Adults Learning Maths forum, much of the writing on adults selects to consider single issues such as work, memory, belief, in isolation and from only one perspective. This research is routed in the belief that it is the interaction of multiple influences that makes the teaching of mature adults unique. New ownership of learning (Knowles, 1980) mixes with ingrained beliefs from the past of what teaching should be (Duffin and Simpson, 2000) and behaviourist desires for measurable change. Life consolidated skills such as instinctive calculation (Fischbein, 1999), use of commonsense mathematics (Coben, 2000) and searching for reasonableness (Benn, 1997) confuse the identification of appropriate teaching hierarchies (Gagne, 1968, 1969, 1985) and 'clash' (D'Ambrosio, 1994) with new behaviours developing from induction into the university Community of Practice (Lave and Wenger, 1999). As discussed in Section 3.1, the cohort of students studied here contained a much higher proportion of mature students than cohorts in many other institutions allowing a unique opportunity to study a large body of mature students together and identify patterns of behaviour that might not otherwise have been recognised, in particular the influence of number on method selection and ideas of non-conservation of operation (Greer, 1994). Further, my own positioning at the vertex of a number of different discourses (university teaching fellow, mathematics teacher, Foundation Year Network member, student mentor and researcher) gave me fresh insights that will be of value to others. My role as Foundation teacher enabled me to pull together these different insights and re-interpret them so that they can impact on the design of learning experience and

curriculum for other Foundation teachers, the majority of whom I believe, are unaware of research in these other domains.

At the end of this current research journey it is apparent that instinctive and intuitive methods right or wrong, recall of number facts correct or corrupted, the ability to chunk information or use automatic processes, the checking of answers and working towards reasonableness, the assumption that all maths is the same, confidence in the method that works, belief in the one right method, and a patchwork of fragmented mathematical memories all come together to form the legacy of learning already present on the brownfield site on which foundation teaching is to build. The message from this work appears to be the importance of making this heritage explicit to both student and tutor, identifying the domains over which previous ideas are valid, exploring the reasons behind certain behaviours and fitting them all into a web of mathematical understanding that can be readily extended into the areas required in future mathematical activity.



## **References**

- Alatorre, S. and Figueras, O. (2005) 'Proportional reasoning of adults with different levels of literacy', in M. Horne and B. Marr, (eds) *Connecting voices in adult mathematics and numeracy: practitioners, researchers and learners. Proceedings of the 12<sup>th</sup> international conference of Adults Learning Maths – A research forum*, Melbourne. pp.42-47.
- Ausubel, D. (1967) 'A cognitive structure theory of school learning', in L. Siegel, (ed) *Instruction: Some contemporary viewpoints*, pp.207-260, San Francisco, Chandler.
- Baddeley, A. (1986) *Working memory*, Oxford, Clarendon Press.
- Bandura, A. (1994) 'Self-Efficacy', in Ramachaudran, (ed) *Encyclopaedia of human behaviour*, vol.4, pp.71-81, New York, Academic Press
- Baroody, A. (2003) 'The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge', in A. Baroody and A. Dowker, (eds) *The development of arithmetic concepts and skills: Constructing adaptive expertise*, Mahwah, NJ, Lawrence Erlbaum Associates.
- Benn, R. (1997) *Adults count too: Mathematics for empowerment*, Leicester, NIACE.
- B.E.R.A. (2004) *Revised Ethical Guidelines for Educational Research*, British Educational Research Association [online]. Available from: <http://www.bera.ac.uk> [Last Accessed 12<sup>th</sup> June 2012].
- Bibby, T. (2002) 'Shame: an emotional response to doing mathematics as an adult and a teacher', *British Educational Research Journal*, vol.28, pp.705-721.

- Bishop, A. (1991) 'Mathematics education in its cultural context', in M. Harris, (ed) *School, Mathematics and Work*, London, Falmer Press.
- Black, P. (1999) 'Assessment, learning Theories and Testing Systems', in P. Murphy, (ed) *Learners, Learning and Assessment*, London, Paul Chapman Publishing.
- Brase, G.L. (2002) "'Bugs" built into the system: How privileged representations influence mathematical reasoning across the lifespan.', *Learning and Individual Differences*, vol.12. pp.391 – 409.
- Bredo, E. (1999) 'Reconstructing Educational Psychology', in P. Murphy, (ed) *Learners, Learning and Assessment*, London, Paul Chapman Publishing.
- Brown, M. (1981a) 'Number operations', in K. Hart, (ed) *Children's Understanding of MATHEMATICS: 11-16*, London, John Murray.
- Brown, M. (1981b) 'Place value and decimals', in K. Hart, (ed) *Children's Understanding of MATHEMATICS: 11-16*, London, John Murray.
- Bruner, J. (1966) *Towards a theory of instruction*, Cambridge, Massachusetts and London, Harvard University Press
- Bryman, A. (2008) *Social Research Methods (3<sup>rd</sup> Edition)*, Oxford, Oxford University Press.
- Buxton, L. (1981) *Do you panic about maths? Coping with maths anxiety*, London, Heinemann Educational Books.
- Carraher, T., Carraher, D., Schliemann, A. (1985) 'Mathematics in the streets and in schools', *British Journal of Developmental Psychology*, vol.3, pp.21-29.
- Clausen-May, T. (2001) *An approach to test development*, Slough, National Foundation for Educational Research.
- Coben, D. (2000 ) 'Mathematics or common sense? Researching 'invisible' mathematics through adults' life histories', in D. Coben, J. O'Donoghue and

- G. Fitzsimons, (eds) *Perspectives on adults learning mathematics: Research and practice*, Dordrecht/Boston/London, Kluwer Academic Publishers.
- Coben, D., Hall, C., Meriel Hutton, B., Sabin, M., Weeks, K. and Woolley, N. (2007) 'Numeracy for nursing. The case for a benchmark', *The changing face of adults mathematics education: learning from the past,, planning for the future. Proceedings of the 14<sup>th</sup> international conference of Adults Learning Maths – A research forum*, University of Limerick, Ireland, pp.88-102.
- Cockcroft, W. (1982) *Mathematics counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*, London, Her Majesty's Stationery Office
- Confrey, J. (1991) 'Learning to Listen: a student's understanding of powers of ten', in E. von Glasersfeld, (ed) *Radical Constructivism in mathematics education*, Dordrecht /Boston /London, Kluwer Academic Publishers.
- Cowan, R. (2003) 'Does it all add up? Changes in children's knowledge of addition combinations, strategies, and principles', in A. Baroody and A. Dowker, (eds) *The development of arithmetic concepts and skills: Constructing adaptive expertise*, Mahwah, NJ, Lawrence Erlbaum Associates.
- D'Ambrosio, U. (1991) 'Ethnomathematics and its place in the history and pedagogy of mathematics', in M. Harris, (ed) *School, Mathematics and Work*, London, Falmer Press.
- D'Ambrosio, U. (1994) 'Ethnomathematics, the Nature of Mathematics and Mathematics Education', in P. Ernest, (ed) *Mathematics, Education and Philosophy An International Perspective*, London, Falmer Press.
- Dawson, (2009) *Introduction to Research Methods* (4<sup>th</sup> Edition), Oxford, How to books ltd.

- Dodd, M. (2007a) Pilot project – unpublished.
- Dodd, M. (2007b) 'The Mathematical Competence of Adults Returning to Learning on a University Foundation Programme: a selective comparison of performance with the CSMS study ' in M. Joubert, (ed) *Informal proceedings of the British society for research into learning mathematics*, University of Manchester, vol.28, pp.15-20
- Dodd, M. (2008) 'CURRENT REPORT: The mathematical competence of adults returning to learning on a university foundation programme: a selective comparison of performance with the CSMS study' in E. Nardi, T. Rowland, L. Haggarty, (eds) *Research in Mathematics education*, Oxford, Routledge, vol. 10, pp.203-204.
- Dowling, P. (1991) 'The contextualizing of mathematics: Towards a theoretical map', in M. Harris, (ed) *School, Mathematics and Work*, London, Falmer Press.
- Draaisma, D. (2004) *Why life speeds up as you get older. How memory shapes our past*, Cambridge, Cambridge University Press.
- Duffin, J. and Simpson, A. (2000) 'Understanding their thinking: the tension between the Cognitive and the Affective' in D.Coben, J. O'Donoghue and G. FitzSimons, (eds) *Perspectives on adults learning mathematics Research and practice*, Dordrecht/Boston/London, Kluwer academic publishers.
- Ebbinghaus, H. (1964) *Memory: a contribution to experimental psychology* (H.Ruger & C.Bussensius, Trans.) New York, Dover. (Original work published 1885)
- Evans, J. (2000) *Adults' Mathematical Thinking and Emotions – A study of numerate practice*, London and New York, Routledge Falmer.

- Filloy, E and Sutherland, R. (1996) 'Designing Curricula for Teaching and Learning Algebra', in A. Bishop, K. Clements, G. Keitel, J. Kilpatrick and C. Laborde, (eds) *International handbook of mathematics education*, Dordrecht/Boston/London, Kluwer Academic Publishers.
- Fischbein, E. (1999) 'Intuitions and Schemata', *Educational Studies in Mathematics*, vol.38, pp.11-50.
- FitzSimons, G. (2005) 'Numeracy and Australian workplaces: Findings and implications', *Australian Senior Mathematics Journal*, vol.19, pp.27-40.
- FitzSimons, G. (2007) 'Adult Numeracy: Crossing Borders of Discourse', in V. Seabright and I. Seabright, (eds) *Crossing borders – Research, reflection and practice in adults learning mathematics. Proceedings of 13<sup>th</sup> International conference of Adults Learning Maths – A research forum*, Queen's University Belfast, Northern Ireland, pp.34-42.
- Gagné, R. (1968) 'Presidential address of division 15 learning hierarchies', *Educational Psychologist*, vol.6:1, pp.1-9.
- Gagné, R. (1969) *The conditions of learning*, London, New York, Sidney, Toronto, Holt, Reinhart and Winston.
- Gagné, R. (1985) *The conditions of learning and theory of instruction (4<sup>th</sup> Edition)*, New York, Reinhart and Winston.
- Gardner, H. (1999) 'Assessment in Context', in P. Murphy, (ed) *Learners, Learning and Assessment*, London, Paul Chapman Publishing.
- Gerdes, P. (1996) 'Ethnomathematics and Mathematics Education', in A. Bishop, K. Clements, G. Keitel, J. Kilpatrick and C. Laborde, (eds) *International handbook of mathematics education*, Dordrecht/Boston/London, Kluwer Academic Publishers.

- Glaser, R. (1999) 'Expert knowledge and processes of thinking', in R. McCormick and C. Paechter, (eds) *Learning and Knowledge*, London, Paul Chapman Publishing
- Gluck, M., Mercado, E., Myers, C. (2008) *Learning and Memory. From brain to behaviour*, New York, Worth Publishers.
- Greeno, J., Pearson, P., Schoenfield, A. (1999) 'Achievement and theories of knowing and learning', in R. McCormick and C. Paechter, (eds) *Learning and Knowledge*, London, Paul Chapman Publishing.
- Greer, B. (1994) 'Extending the meaning of multiplication and division', in G. Harel and J. Confrey, (eds) *The development of multiplicative reasoning in the learning of mathematics*, Albany, State University of New York Press.
- Harel, G., Behr, M., Post, T. and Lesh, R. (1994) 'The Impact of number type on the solution of multiplication and division problems: Further Investigations', in G. Harel and J. Confrey, (eds) *The development of multiplicative reasoning in the learning of mathematics*, Albany, State University of New York Press
- Hart, K., (ed) (1981a) *Children's Understanding of MATHEMATICS: 11-16*, London, John Murray.
- Hart, K. (1981b) 'Ratio and proportion' in K. Hart, (ed) *Children's Understanding of MATHEMATICS: 11-16*, London, John Murray.
- Hart, K. (1981c) 'Hierarchies in Mathematics Education', *Educational Studies in Mathematics*, vol.12, no.2, pp.205-218.
- Hiebert, J. and Lefevre, P. (1986) 'Conceptual and procedural knowledge in mathematics: An introductory analysis.' in J. Hiebert , (ed.) *Conceptual and procedural knowledge: The case of mathematics*, Hillsdale, NJ:Lawrence Erlbaum Associates.
- Just, M. and Carpenter, P. (1992) 'A capacity theory of comprehension:

- Individual differences in working memory', *Psychological Review*, vol.98, pp.122–149.
- Kahneman, D. (2002) 'Maps of bounded rationality: A perspective on intuitive judgment and choice', in T. Frangsmyr (ed.), *Les Prix Nobel*, pp.416–499.
- Kaput, J. and West, M. (1994) 'Missing-value proportional reasoning problems: 'Factors affecting informal reasoning patterns', in G. Harel and J. Confrey, (eds) *The development of multiplicative reasoning in the learning of mathematics*', Albany, State University of New York Press.
- Karsenty, R. (2002) 'What do adults remember from their high school mathematics? The case of linear functions', *Educational Studies in Mathematics*, vol.51, pp.117-144.
- Karsenty, R. (2004) 'Mathematical self-schema: a framework for analyzing adults' retrospection on high school mathematics', *The Journal of Mathematical Behavior*, vol.23, pp.325-349.
- Kirk, P. and Ashcraft, M. (2001) 'Telling Stories: The perils and promise of Using Verbal Reports to Study Maths Strategies', *Journal of experimental psychology/learning, Memory and Cognition*, vol.27, pp.157-175.
- Klinger, C. (2007) 'Experience, the difference: Maths attitudes and beliefs in commencing undergraduate students and pre-tertiary adult learners' in *The changing face of adults mathematics education: learning from the past , planning for the future. Proceedings of the 14<sup>th</sup> international conference of Adults Learning Maths – A research forum*, University of Limerick, Ireland. pp.197–207.
- Knowles, M. (1980) *The modern practice of adult education: From Pedagogy to Androgogy (2<sup>nd</sup> Edition)*, Englewood Cliffs, Prentice Hall,/ Cambridge.

- Kuchemann, D. (1981) 'Positive and negative numbers', in K. Hart, (ed) *Children's Understanding of MATHEMATICS: 11-16*, London, John Murray.
- Lamon, S. (1994) 'Ratio and proportion: Cognitive foundations in unitising and norming', in G. Harel and J. Confrey, (eds) *The development of multiplicative reasoning in the learning of mathematics*, Albany, State University of New York Press.
- Lannin, J., Barker, D. and Townsend, B. (2007) 'How students view the general nature of their errors', *Educational Studies in Mathematics*, vol.66, pp.43-59.
- Lave, J. (1988) *Cognition in Practice: Mind, mathematics and culture in everyday life*, Cambridge, Cambridge University Press.
- Lave, J. and Wenger, E. (1999) 'Legitimate Peripheral Participation in Communities of practice', in R. McCormick and C. Paechter, (eds) *Learning and Knowledge*, London, Paul Chapman Publishing.
- LeFevre, J., Smith-Chant, B., Hiscock, K., Daley, K., Morris, J. (2003) 'Young adults' strategic choices in simple arithmetic: Implications for the development of mathematical representations', in A. Baroody and A. Dowker, (eds) *The development of arithmetic concepts and skills: Constructing adaptive expertise*, Mahwah, NJ, Lawrence Erlbaum Associates.
- Leron, U. and Hazzan, O. (1997) 'The world according to Johnny: A coping perspective in mathematics education', *Educational Studies in Mathematics*, vol.32, pp.265-292.
- Leron, U. and Hazzan, O. (2006) 'The rationality debate: Applications of cognitive psychology to mathematics education', *Educational Studies in Mathematics*, vol.62, pp.105-126.
- Lithner, J. (2008) 'A research framework for creative and imitative reasoning', *Educational Studies in Mathematics*, vol.67, pp.255-276.



- Maier, E. (1991) 'Mathematics education in its cultural context', in M. Harris, (ed) *School, Mathematics and Work*, London, Falmer Press.
- Maslow, A. (1970) *Motivation and Personality (2<sup>nd</sup> Edition)*, New York, Harper Collins.
- Merriam, S., Caffarella, R., Baumgartner, L. (2007) *Learning in adulthood, a comprehensive guide (3<sup>rd</sup> Edition)*, San Francisco, Jossey-Bass.
- Nunes, T., Schliemann, A., and Carraher, D. (1993). *Street Mathematics and School Mathematics*. Cambridge, Cambridge University Press.
- Piaget, J. and Inhelder, B. (1958) *The Growth of Logical Thinking from Childhood to Adolescence*, New York, Basic Books.
- Pozzi, S., Noss, R., Hoyles, C. (1998) 'Tools in Practice, Mathematics in use', *Educational Studies in Mathematics*, vol.36, pp.105-122.
- Rogers, C. (1983) *Freedom to Learn for the Eighties*, Columbus, OH: Charles Merrill.
- Roth, W. (1999) 'Authentic School Science: Intellectual Traditions', in R. McCormick and C. Paechter, (eds) *Learning and Knowledge*, London, Paul Chapman Publishing.
- Safford, K. (2000) 'Algebra for Adult Students: the Student Voices', in D.Coben, J. O'Donoghue and G. FitzSimons, (eds) *Perspectives on adults learning mathematics Research and practice*, Dordrecht/Boston/London, Kluwer academic publishers.
- Sacks, O. (1999) 'Making up the mind', in P. Murphy, (ed) *Learners, Learning and Assessment*, London, Paul Chapman Publishing.
- Schoenfeld, A. (1985) *Mathematical problem solving*, New York, Academic Press.
- Schoenfeld, A. (1992) 'Learning to think mathematically: Problem solving,

- metacognition, and sense-making in mathematics', in D. Grouws, (ed.) *Handbook for Research on Mathematics Teaching and Learning*, pp.334-370, New York, MacMillan.
- Sewell, B. (1981), *Use of Mathematics by Adults in Daily Life*, ACACE
- Siegler, R. (1988) 'Individual differences in strategy choices: Good students, not-so-good students, and perfectionists', *Child Development*, vol.59, pp.833-851.
- Simon, B. (1999) 'Why No Pedagogy in England?', in J. Leach and B. Moon, (eds) *Learners and Pedagogy*, London, Paul Chapman Publishing.
- Skemp, R. (1979) *The Psychology of Learning Mathematics*, Middlesex, Pelican Books.
- Skinner, B. (1971) *Beyond freedom and dignity*, New York, Knopf.
- Stanovitch, K. and West, R. (2000) '*Individual differences in reasoning: Implications for the rationality debate?*', *Behavioral and Brain Sciences*, vol.23, pp.645-726.
- Stavy, R. And Tirosh, D. (2000) *How Students (Mis-)Understand Science and Mathematics: Intuitive Rules*, Columbia, Teachers College Press.
- Strauss, A. and Corbin, I. (1998) *Basics of Qualitative Research: techniques and procedures for developing grounded theory*, 2<sup>nd</sup> edition, London, Sage.
- Swan, M. (2001) 'Dealing with misconceptions in mathematics', in P. Gates, (ed) *Issues in Mathematics teaching*, London, Routledge Falmer.
- Swan, M. (2006) *Collaborative learning in Mathematics, a challenge to our beliefs and practices*, Leicester, NIACE.
- The Open University (2001) *Research Methods in Education Handbook*, Milton Keynes, The Open University.

- Thorndike, E.L. (1927) 'The law of effect', *American Journal of Psychology*, vol.39, pp.212-222
- von Glasersfeld, E. (1995) *Radical Constructivism, A Way of Knowing and Learning*, London, Routledge Falmer.
- Vygotsky, L. (1978) *Mind and Society: The development of higher psychological processes*, Cambridge, Harvard University Press.
- Walen, S, and Williams, S. (2002) 'A matter of time: Emotional responses to timed mathematics tests', *Educational Studies in Mathematics*, vol.49, pp.361-378.
- Wedge, T. (2000) 'Technology, Competencies and Mathematics' in D.Coben, J. O'Donoghue and G. FitzSimons (eds) *Perspectives on adults learning mathematics Research and practice*, Dordrecht/Boston/London, Kluwer academic publishers.
- Williams, S. (1999) 'Mathematics (grades 7-12)' in R. McCormick and C. Paechter,.(eds) *Learning and Knowledge*, London, Paul Chapman Publishing.
- Wood, D., Bruner, S.J. and Ross, G. (1976) 'The role of tutoring in problem solving', *Journal of Child Psychology*, vol.17, pp.89-100, Oxford, Pergamon Press.

# Appendix 1 – Activities

## Appendix 1a – Questionnaire – Part A (Non-calculator)

October 07

The following questions have been designed to find out about your experiences with, skills in and feelings about numbers or 'maths'.

The results for the group as a whole will be used in helping this year's courses work better, and for further studies by several members of staff aimed at making maths teaching more appropriate in general.

There is no way that your work can be attributed to you, so please be as honest as you can about what you do and do not know and whether something is a guess.

### DO NOT WRITE YOUR NAME ON ANY OF THE SHEETS

To help cross-match at a later date

What was the name of your primary school?

What was your mother's first name?

Were you taught in English (or Welsh) at school Yes / No

Based on ideas from Hart, K., (ed) (1981a) *Children's Understanding of MATHEMATICS: 11-16*, London, John Murray.

Please ring the letter for your progression route

- A Business, Business Finance, Economics, Accounting
- B Biological Sciences, Biomedical Sciences, Medicine
- C Engineering, Chemistry, Computer Sciences
- D Earth Sciences, Geology
- E Primary Education
- F Applied psychology, Human Sciences, Biological Anthropology, Medical Anthropology, Sociology, Criminology
- G Other \_\_\_\_\_

Please ring any of the maths qualifications below that you have, filling in any gaps if appropriate

NONE A-Level (grade \_\_\_\_ ) As-Level (grade \_\_\_\_ )  
CSE Degree O-Level (grade \_\_\_\_ )  
GCSE (grade \_\_\_\_ ) Other \_\_\_\_\_

Please ring when you last studied maths (excluding study on the Foundation Programme or summer school)

Within the last 2 years Between 2 and 5 years ago  
Between 5 and 10 years ago More than 10 years ago.

Please ring the value that best relates to your feelings about maths study this year

very very not at all  
anxious anxious  
1 2 3 4 5 6 7 8 9 10

Please ring the value that best relates to your feelings about overall study on the Foundation Programme this year

very very not at all  
anxious anxious  
1 2 3 4 5 6 7 8 9 10

After you have answered a question, please could you put a cross in the column that best describes how confident you are about your answer.

For each of the questions 1-5, ring the CALCULATION (or calculations) that you think you would have to do. (Do not actually work it out)  
You may sometimes choose to select more than one answer.

	Very confident	confident	not very confident	Just guessed
<b>Question 1</b> A bar of chocolate contains 20 squares. There are 5 squares in a row. How do you work out how many rows there are? $20 \div 5$ $5 \times 4$ $20 \times 5$ $5 - 20$ $10 \div 10$ $20 \div 5$ $20 - 5$ $5 + 20$				
<b>Question 2</b> A cafe makes sandwiches. You can choose from 3 sorts of bread and 4 sorts of filling. How do you work out how many different sandwiches you could choose? $3 \times 4$ $4 - 3$ $4 + 3$ $3 - 4$ $12 \div 3$ $4 \div 3$ $4 \times 3$ $3 + 3$				
<b>Question 3</b> My old car travels at 43.8 miles per gallon. How do you work out how far it would travel with 7.4 gallons? $43.8 - 7.4$ $7.4 \times 43.8$ $43.8 + 7.4$ $43.8 \div 7.4$ $43.8 \times 7.4$ $7.4 \div 43.8$ $7.4 - 43.8$ $7.4 + 43.8$				
<b>Question 4</b> Potatoes cost 78.4 p per kg. How would you work out how much 0.52 kg of potatoes cost? $78.4 + 0.52$ $0.52 + 78.4$ $0.52 - 78.4$ $78.4 \times 0.52$ $78.4 \div 0.52$ $0.52 \times 78.4$ $0.52 \times 78.4$ $78.4 - 0.52$				
<b>Question 5</b> A college has 399 students to be allocated to 19 tutors. Each tutor is to be allocated the same number of students. How do you work out how many students each tutor has? $399 - 19$ $19 + 399$ $19 - 399$ $399 \times 19$ $399 \div 19$ $19 \div 19$ $19 \times 21$ $399 \div 19$				

For each of the questions 6-13, ring the correct answer (or answers)

	Very confident	confident	not very confident	Just guessed
<b>Question 6</b> What is 25% as a fraction? $\frac{1}{25}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{25}{100}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{1}{2}$ other				
<b>Question 7</b> Which calculation gives the bigger answer? $6 \times 3$ or $6 + 3$				
<b>Question 8</b> Which number is nearest in size to the answer of $40 \div 200$ 50      5      0.5      0.05 20      2      0.2      0.02				
<b>Question 9</b> Which calculation gives the bigger answer? $8 \times 0.4$ or $8 \div 0.4$				
<b>Question 10</b> Which number is nearest in size to 0.28 0.2      20      0.3      30      0.02      3				
<b>Question 11</b> Which calculation gives the bigger answer? $0.5 \times 0.2$ or $0.5 \div 0.2$				
<b>Question 12</b> Which is the bigger number:      30 100      or      30 095				
<b>Question 13</b> Which is the bigger number      7.06      or      7.5				

Try to answer as many of the following questions as you can and show any working or method that you use. You may not use a calculator but if you cannot work something out without one, just show what you would have calculated.

	Very confident	confident	not very confident	Just guessed
<p>Question 14</p> <p>Three snakes are kept in captivity and fed on mice. The number of mice they are given depends on their length.</p> <p>A _____ 50cm _____ 100cm _____ 150cm</p> <p>B _____</p> <p>C _____</p> <p>a) If snake A is fed 2 mice.</p> <p>(i) How many would snake B get?</p> <p>(ii) How many would snake C get?</p> <p>b) If snake B is fed 6 mice, how many should snake C get?</p> <p>c) If snake C is fed 12 mice, how many should snake B get?</p>				
<p>Question 15</p> <p>What is <math>7.75 + 10</math></p>				
<p>Question 16</p> <p><math>3 + 6 =</math></p>				

	Very confident	confident	not very confident	Just guessed
<p>Question 17</p> <p>What is 10% as a fraction?</p>				
<p>Question 18</p> <p>8 cards cost 45p. How much would 12 cost?</p>				
<p>Question 19</p> <p>In a group of 600 mice, 42 have blue eyes. What percentage is this?</p>				
<p>Question 20</p> <p>A caterer has a recipe for 80 people.</p> <p>It requires: 12 eggs 2 lemons <math>\frac{1}{2}</math> box of packet mix</p> <p>For 40 people</p> <p>How many eggs would you need?</p> <p>How many lemons would you need?</p> <p>How much packet mix would you need?</p> <p>For 60 people</p> <p>How many eggs would you need?</p> <p>How many lemons would you need?</p> <p>How much packet mix would you need?</p>				

Question 21 6% of a group of adults carry a particular disease. If there are 250 people in the group, how many carry the disease?	Very confident				
Question 22 A picture which originally cost £20 is reduced by 5% in a sale. What is its sale price?	confident				
Question 23 A caterer has another recipe which requires him to use 20 spoonfuls for 8 people.  How much for 15 people?	not very confident				
Question 24 What is 43% of 800	Just guessed				
Question 25 24 birds in a colony of 300 have inherited a particular characteristic. What percentage is this?					

Finish Time \_\_\_\_\_

Thank you for completing this set of questions.  
Only go on to the next booklet if you are happy that you have done as much as you possibly can in this booklet.

## Appendix 1b – Questionnaire – Part B (Calculator)

### PART B

Please only do this booklet if you have completed the first booklet. Do not go back to the first booklet if you suddenly remember how to do something. You may use a calculator.

Please work through the questions in order and do not go back if you change your mind or a later question triggers a memory. It is unlikely that you will get through all the questions in the time we have set aside for this questionnaire so don't worry about the clock or the questions you don't get to.

Try to answer as many of the following questions as you can and show any working or method that you use. If you use a calculator show what you put into it.

After you have answered a question, please could you put a cross in the column that best describes how confident you are about your answer.

	Very confident	confident	not very confident	Just guessed
Question 1 What can you say about $y$ if $y + 4 = 9$				
Question 2 What is 59% of £600				
Question 3 What can you say about $a$ if $a = b + 3$ and $b = 2$				
Question 4 A small packet of mix contains 120g and costs 96p. A large packet of mix contains 200g and costs 170p.  Which is the better value? Show any working to explain your decision.				
Question 5 The value of a flat increases by 4% per year. If it is valued at £20 800 this year, how much was it worth last year?				



	Very confident	confident	not very confident	Just guessed
Question 6 6 cards cost 48p. How much would 11 cost?				
Question 7 add 3 onto $h + 4$				
Question 8 In a college of 400 people, 75 study biology. What percentage is this?				
Question 9 If $a + b = 61$ , what is $a + b + 2$				
Question 10 $2d - k + d =$				
Question 11 If $m - 124 = 341$ what is $m - 125$				
Question 12 Which is larger, $2c$ or $c + 2$				
Question 13 What can you say about $a$ if $a = 2b + 5$ and $b = 3$				

	Very confident	confident	not very confident	Just guessed
Question 14 If $a + b = 6$ , what is $a + b + c$				
Question 15 Multiply $n + 3$ by 5				
Question 16 What can you say about $a$ if $a = b + c$ and if $a + b + c = 20$				
Question 17 What can you say about $d$ if $d + e = 10$ and $d$ is less than $e$				

## Appendix 1c – Questionnaire – Attitude Section

For each of the following items please indicate to what extent you would generally feel either relaxed or anxious in the situations they describe. Please rate the situation according to your immediate feelings, on the following scale:

1. I would be very relaxed
2. I would be relaxed
3. I would be fairly relaxed
4. I would be neither relaxed nor anxious
5. I would be a little anxious
6. I would be moderately anxious
7. I would be very anxious

	1 very relaxed	2 relaxed	3 fairly relaxed	4 neither relaxed nor	5 a little	6 moderately anxious	7 very anxious
1. Determining the amount of change you would get from a purchase of several items							
2. Asking a stranger which bus to catch in a strange town.							
3. Coming for a Foundation Programme interview							
4. Calculating which is the cheapest method of getting somewhere by public transport							
5. Walking into the room for the pre-foundation session last week (if you went)							
6. Dividing a two digit number by a five digit number in private with pencil and paper							
7. Choosing an item of clothing							
8. Finding out you needed to do maths on the Foundation Programme							
9. Reading your P60 (or other statement) showing your annual earnings and taxes							
10. Going to your first IT session next week							
11. Giving a spoken presentation to other students							
12. Walking into the room on the first day of induction							
13. Deciding which film to go and see by yourself							
14. Sitting in this maths class and waiting for the tutor to arrive?							
15. Being asked a question by a tutor in an English class							
16. Reading a cash register receipt after buying something							
17. Asking a question in a maths class							
18. Doing a maths exam							
19. Doing an English exam							
20. Climbing a ladder							
21. Listening to a lecture in a maths class							
22. Being asked a question by a tutor in an English class							
23. Talking to a group of strangers (people from a similar social background to yourself unknown to you)							
24. Completing this questionnaire							

Based on ideas from Evans, J. (2000) *Adults' Mathematical Thinking and Emotions – A study of numerate practice*, London and New York, Routledge Falmer.

## Appendix 1d – Mathematics problems used in interview (many from CSMS survey).

1. A bar of chocolate is broken into 12 squares.

There are 3 squares in a row.

How do you work out how many rows there are?

$$12 \div 3 \qquad 3 \times 4 \qquad 12 \times 3 \qquad 3 - 12$$

$$6 + 6 \qquad 12 \div 3 \qquad 12 - 3 \qquad 3 \div 12$$

(Brown, 1981a, p.24)

2. In each pair, ring the one which gives the BIGGER answer.

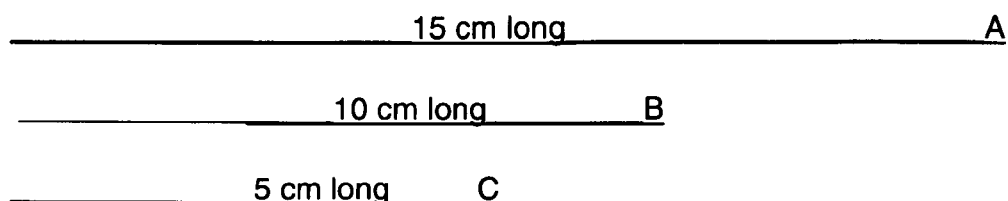
a)  $8 \times 4$  or  $8 \div 4$

b)  $8 \times 0.4$  or  $8 \div 0.4$

c)  $0.8 \times 0.4$  or  $0.8 \div 0.4$  (Brown, 1981b, p.54)

3. What is  $\frac{2}{3}$  of 12 ?

4. There are three eels A, B and C in the tank at the zoo.



The eels are fed sprats, the number depending on their lengths.

If C is fed two sprats, how many sprats should:

- (i) B and (ii) A be fed to match?

If B eats 12 sprats, how many sprats should A be fed to match?

If A gets 9 sprats, how many sprats should B get to match?

(Hart, 1981b, p. 90)

5. Which is bigger  $0.8 \times 1.2$  or  $0.8 \div 1.2$  ?

6. 6 % of children in a school have free dinners.

There are 250 children in the school.

How many children have free dinners?

(Hart, 1981b, p. 96)

7. The newspaper says that 24 out of 800 Avenger cars have a faulty engine.

What percentage is this? (Hart, 1981b, p. 96)

8. The price of a coat is £20. In a sale it is reduced by 5%.

How much does it cost now? (Hart, 1981b, p. 96)

9. What is  $\frac{3}{4}$  of 12 ?

## Appendix 1e – Class Task used for sub-cohort

Figure A1.1: *Class task given to sub-cohort*

*You may use a calculator*

**Potatoes cost 78.4 p per kg.**

**How much would 0.52 kg of potatoes cost?** (Just write down your answer)

**What calculation (s) did you do?**

**Did you try any other calculations first?**

## Appendix 1f – Extracts from Notes used for Question Selection

### ***RATIO***

#### Evans' study

No ratio in paper test except cost of 8 items at 14p each and 1 item if 5 cost 65p.

Try using a combination question – more likely to use unitary method?

Best buy question in interviews. Lots of other people have used best buy problems. Evans uses large bottle 30 oz 69p and small bottle 20oz 52p from

Sewell. ***Put a best buy problem in. Use different values  $\frac{3}{5}$  not  $\frac{2}{3}$***

#### CSMS study

##### *Recipe questions*

There is a very wide difference in facility. This is something that adults will be doing in everyday life. This is also useful in the Genetics work they are about to meet. ***Definitely use***

1) Original question: Given amount for 8 find amount for 4, Given amount for 8 find the amount for 6, Three amounts, easy whole numbers, 2, 1/2 pint cream. The analysis discusses building up by adding the amounts for 2 and 6. Does the way that the questions were asked influence this? What would happen if they weren't consecutive questions? ***Try asking in different places in the questionnaire***

2) The recipe questions only use halving and doubling strategies. ***Ask one which cannot be solved this way***

##### *Eel Questions*

Based on Piaget. Large facility. ***Use***

Original Questions: discrete objects ratio 1:2:3 (same as Piaget) used 5, 10 and

15cm. 2 fed to 5cm-find 10 & 15(x2 and x3), 12 fed to 10 find 15(x  $1\frac{1}{2}$ ), 9 fed to

the 15 find  $10 \left( \times \frac{2}{3} \right)$ , continuous quantity ratio 2:3:5 used 10, 15, 25 cm fishfingers

***1) Do eels eat sprats? Are eels most appropriate context for***

## ***OPERATIONS and PLACE VALUE and DECIMALS***

Evans's study - not really tested

### CSMS study

Lots of lovely questions but some too trivial? All fairly high facilities.

Actually not high facilities – allowed inversions. Is it worth testing?

Other Foundation tutor keen to use some of these. Nice lead in?

All multiple choice which is also nice.

### *Questions*

Stories for +, -, x, ÷ then ring the operation

### *Division*

12 squares in chocolate, 3 in row, how many rows (facilities 71,78,84) less inversions.

Larger numbers play? (facilities almost same)

391 daffodils to be planted in 23 flowerbeds (72, 81, 82) but 47 and 33 inversions.

Interesting work on inversions Include daffodil equivalent.

### *Subtraction*

Smaller numbers (91,93,93)

261 miles from London to Leeds. travel 87 miles, how much further (79,83,91)

### *Addition*

Signpost (all ages similar in 70s) problem with understanding context exposed.

Shouldn't trouble adults?

### *Multiplication*

Make sandwiches, 3 sorts of bread and 6 sorts of fillings (46,62,74)

Table 92.3 cm. How many inches (1 inch is 2.54 cm)

Beef is 88,2 pence per Kg cost of 0.58g beef? number less than 1 (18,17,21,29)  
41.8 miles per gallon. How many miles on 8.37 gallons? higher more than 1 (32,  
42,54,53) Try some

*Decimal questions and Place value*

Just use a few. revisit later

Ring the number nearest in size to  $59 \div 190$  (not work out sum)  
.003/.03/.3/30/300/3000

Very low facilities 15 – 22 always more inversions **USE**

Ring the one which gives the bigger answer

$8 \times 4$  or  $8 \div 4$ , similar 8 and 0.4, similar 0.8 and 0.4 **USE**

Ring the bigger number: 20 100 or 20 095 (86 -94)

Ring the bigger number 4.06 or 4.5 (66-80)

Ring the number nearest in size to 0.18 0.1/10/0.2/20/0.01/2 (44-59)

Lots more on decimals but enough for now  
science?

***Use snakes eating mice and 50, 100, 150***

*2) Adults know fishfingers come in standard sizes. Need better context.*

***Probably do not include in this short pilot***

*Question adapted from Karplus. More complex. **No space to include on test***

*Question on chemical quantities*

A 'level 4' question. Bears some similarity to things some of the students will be doing in chemistry.

Original Question: Metal alloy, 1 part Hg to 5 parts Cu, 3 parts tin to 10 parts Cu,  
8 parts Zn to 15 parts Cu parts of Hg to parts of tin and parts of Zn to parts of tin

***Include***

## ***OPERATIONS and PLACE VALUE and DECIMALS***

Evans's study - not really tested

### CSMS study

Lots of lovely questions but some too trivial? All fairly high facilities.

Actually not high facilities – allowed inversions. Is it worth testing? Other

Foundation tutor keen to use some of these. All multiple choice. Nice lead in?.

Stories for +, -,  $\times$ ,  $\div$  then ring the operation

### *Division*

12 squares in chocolate, 3 in row, how many rows (facilities 71,78,84) less inversions.

Larger numbers play? (facilities almost same)

391 daffodills to be planted in 23 flowerbeds (72, 81, 82) but 47 and 33 inversions.

Interesting work on inversions ***Include daffodil equivalent.***

### *Subtraction*

Smaller numbers (91,93,93)

261 miles from London to Leeds. travel 87 miles, how much further (79,83,91)

### *Addition*

Signpost (all ages similar in 70s) problem with understanding context exposed.

Shouldn't trouble adults?

### *Multiplication*

Make sandwiches, 3 sorts of bread and 6 sorts of fillings (46,62,74)

Table 92.3 cm. How many inches (1 inch is 2.54 cm)

Beef is 88.2 pence per Kg cost of 0.58g beef? number less than 1 (18,17,21,29)

41.8 miles per gallon. How many miles on 8.37 gallons? higher more than 1 (32,

42,54,53) ***Try some***

### *Decimal questions and Place value*



## Appendix 2 - The Students

### Appendix 2a – Student Information

Table A2.1: Number of Foundation Students (2006 and 2007 Cohorts)

	Home Students	International Students (Not part of study)
2006	78	15
2007	79	31

#### ***Age and Gender (Home Students)***

Figure A.2.1: Graph to show Ages of Foundation Cohort (2006)

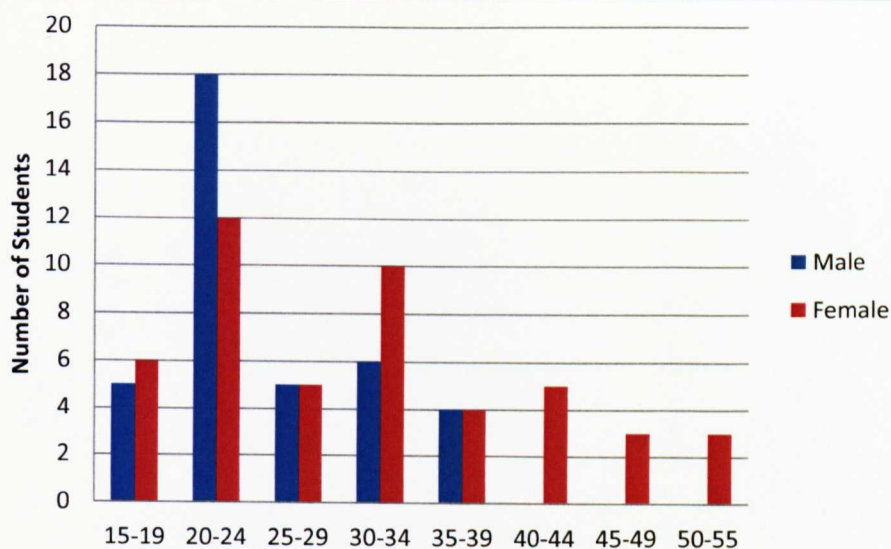
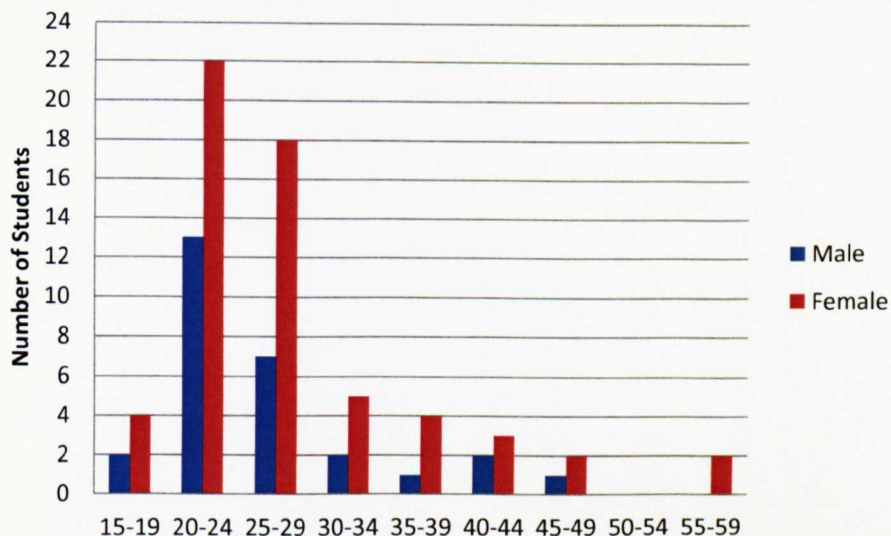


Figure A.2.2: Graph to show Ages of Foundation Cohort (2007)



## Progression Route and Gender (Home Students)

Figure A.2.3: Graph to show Progression Routes for Foundation Cohort (2006)

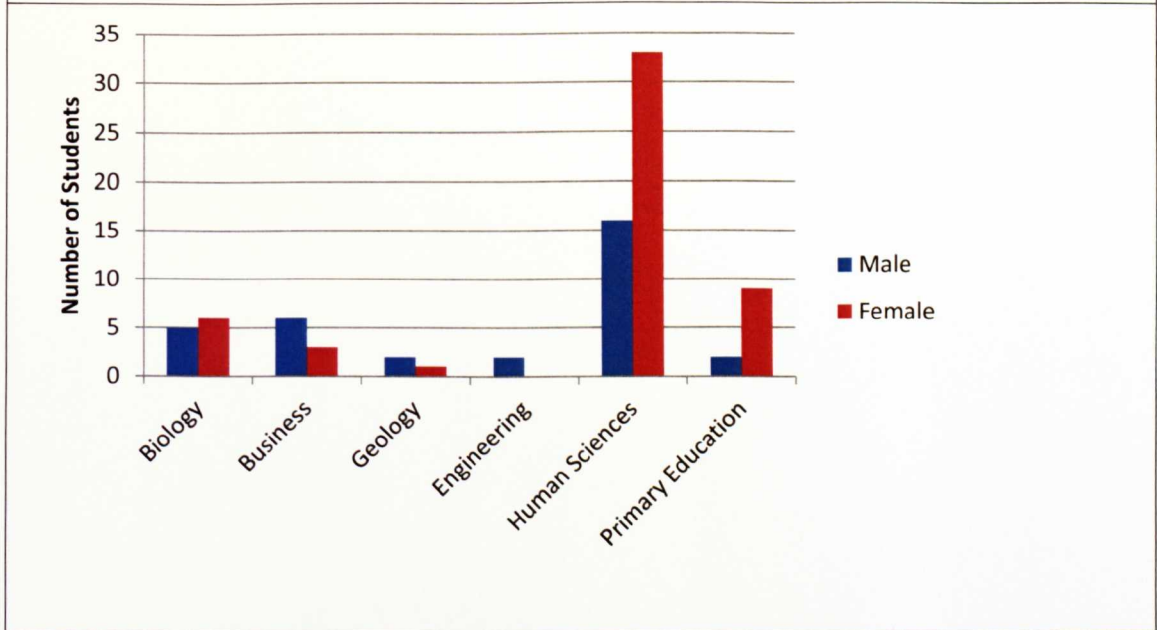
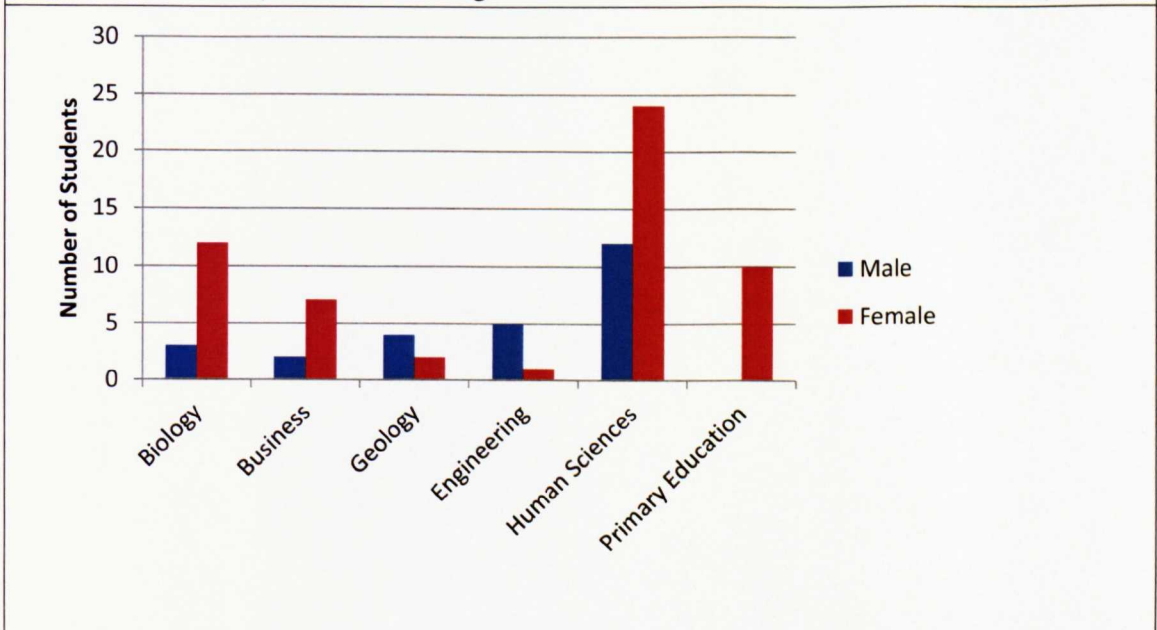


Figure A.2.4: Graph to show Progression Routes for Foundation Cohort (2007)



## Highest Mathematics Qualification (Home Students)

Figure A.2.5: Graph to show Highest Mathematics Qualifications of Students Based on Questionnaire Declarations. (2006)

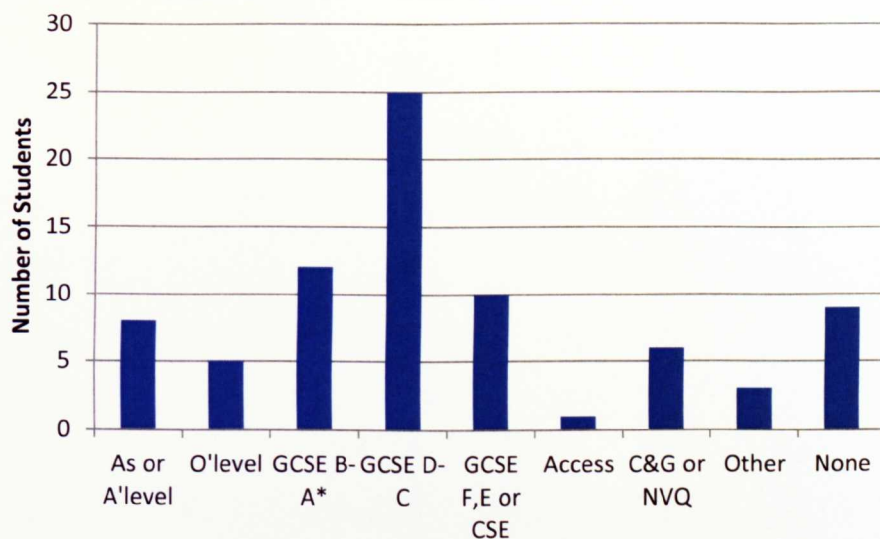
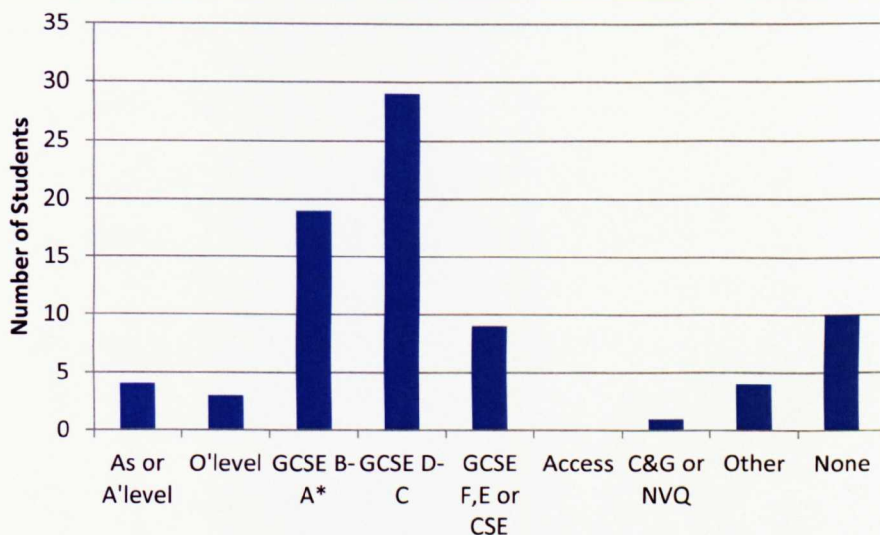


Figure A.2.6: Graph to show Highest Mathematics Qualifications of Students Based on Questionnaire Declarations. (2007)



## **Appendix 2b – The Students Interviewed**

Anxiety ratings: 1 – Very anxious, 10 – Not at all anxious

### ***Ann - Route – Human Science.***

Last studied maths more than 10 years ago.

Highest maths qualification – CSE

Anxiety about maths study at beginning – rating 2

Anxiety about maths study at follow-up – rating 4

Anxiety about general foundation study at beginning – rating 4

Anxiety about general foundation study at follow-up – rating 7

First score 28 / 34                  Follow-up score 25.5 / 34

### ***Brenda- Route – Biomedical Sciences***

Last studied maths between 5 and 10 years ago.

Highest maths qualification – GCSE grade C

Anxiety about maths study at beginning – rating 2

Anxiety about maths study at follow-up – rating 4

Anxiety about general foundation study at beginning – rating 7

Anxiety about general foundation study at follow-up – rating 9

First score 24 / 34                  Follow-up score 27 / 34

### ***Christine- Route – Primary Education.***

Last studied maths between 2 and 5 years ago.

Highest maths qualification – GCSE grade D

Anxiety about maths study at beginning – rating 8

Anxiety about maths study at follow-up – rating 8

Anxiety about general foundation study at beginning – rating 5

Anxiety about general foundation study at follow-up – rating 5

First score 21.5 / 34                  Follow-up score 27 / 34

### ***Diane - Route – Applied Psychology***

Last studied maths between 5 and 10 years ago.

Highest maths qualification – GCSE grade C

Anxiety about maths study at beginning – rating 7

Anxiety about maths study at follow-up – rating 8

Anxiety about general foundation study at beginning – rating 5

Anxiety about general foundation study at follow-up – rating 5

First score 28 / 34                  Follow-up score 28 / 34

### **Elaine - Route – Human Science.**

Last studied maths more than 10 years ago.

Highest maths qualification – None

Anxiety about maths study at beginning – rating 1

Anxiety about maths study at follow-up – rating 1

Anxiety about general foundation study at beginning – rating 5

Anxiety about general foundation study at follow-up – rating 5

First score 15 / 34                  Follow-up score 16.5 / 34

## Appendix 3 - Questionnaire Responses

### Appendix 3a - October 2007

#### *Facilities for questions and some common incorrect answers*

Table A3.1: Adult <i>Facilities for questions and some common incorrect answers</i> (Oct. 2007)					
			Number	%	Confidence Rating
q.1	Chocolate squares	correct division	62	78	8.6
		reverse division	6	8	
q.2	sandwiches	either or both x	71	90	7.8
q.3	gallons&miles	either or both x	47	59	7.6
		division or div &x	28	35	
q.4	potatoes	either or both x	27	34	6.6
		division or div &x	17	22	
q.5	college tutors	correct division	72	91	8.1
		reverse or both divisions	5	6	
q.6	25 percent as fraction	one quarter	52	66	8.8
		25/100	7	9	
		both	15	19	
		one 25th	2	3	
q.7	bigger 6x/3	correct x	78	99	9.2
q.8	40/200	correct	28	35	6.9
		incorrect place	14	18	
		reverse div	37	47	
q.9	bigger 8x/0.4	correct /	41	52	7
q.10	round 0.28	correct	67	85	8.1
		incorrect 0.2	3	4	
q.11	bigger 0.5x/0.2	correct	32	41	6.4
Combined	Bigger patterns	correct x//	25	32	
		x/x	15	19	
		xxx	30	38	
		xx/	7	9	
q.12	30100 or 31095	correct	78	99	8.8
q.13	7.06 or 7.5	correocr	74	94	8.6
q.14ai	snakes x 2	correct	78	99	8.1
q.14aii	snakes x 3	correct	68	86	7.9
		inc 5	7	9	
q.14 b	snakes x 1 1/2	correct	63	80	7.8
		inc 8	10	13	
q.14c	snakes x 2/3	correct	55	70	7.3
		inc 9	6	8	
		inc 6	7	9	
		inc 10	7	9	
q.15	7.75/10	correct	55	70	6
		inc 0.75	5	6	
		inc77.5	4	5	
		inc 1	4	5	



			Number	%	Confidence Rating
q.16	Negative nos	correct -3	55	70	5.9
q.17	10 percent as fraction	one tenth	58	73	7.2
		ten hundredths	6	8	
		both above or 0.1	9	11	
q.18	cost of 12 cards	correct	60	76	7.5
		inc 12x8	3	4	
		inc12x48	2	3	
q.19	42 out of 600 as percent	correct	36	46	4.9
		inc 600/42	9.5	12	
		inc 42x6 var	2.5	3	
q.20.ai	recipe 1/2of 12	correct	79	100	8.5
q.20aaii	recipe 1/2of 2	correct	78	99	8.6
q.20aiiii	recipe 1/2 of 1/2	correct	74	94	8.2
q.20bi	recipe 3/4 12	correct	54	68	7.6
		inc 8	16	20	
		inc 10	3	4	
q.20bii	recipe 3/4 of 2	correct	67	85	7.4
q.20biii	recipe 3/4 of 1/2	correct	15	19	5.5
		inc 1/3	21	27	
		inc 2/3	8	10	
q.21	6% of 250	correct and/or method	42	53	5.6
		inc var 6/25 or 25/6	10	13	
q.22	£20 reduced by 5%	correct	50	63	6.8
		reduction only or method	7	9	
		inc £16	5	6	
		inc £15	4	5	
		inc £19.50	4	5	
q.23a	recipe 3/4 of 20	correct and/or method	50	63	6
		inc 14	5	6	
		inc 16	6	8	
		inc 17	3	4	
		inc 18	4	5	
q.23b	recipe 20/8 x 15	Correct and/or method	37	47	5.4
		rounding 37 or 38	7	9	
		inc 35	2	3	
		inc 36	3	4	
		inc40	2	3	
q.24	43% of 800	correct and/or method	39	49	4.7
		inc var 43/800 or 800/43	4	5	
		near? 345-395	8	10	
		near? 320-340	3	4	
q.25	24 out of 300 as %	correct and/or method	37	47	4.8
		inc var 24x3	10	13	
		inc var 300/24	10	13	
	both complex snakes correct		50	63	
	first correct and second answer 9		6	8	
	both incorrect by ad or subtract 2		6	8	

## Appendix 3b - October 2006

**Topic based facilities for questions and some common incorrect answers.**

**Table A3.2: Results for questions on Operation Selection. (Brown, 1981a)**

		Student Survey	CSMS Survey	CSMS Age
chocolate bar, how many rows	Correct division	65%	32%	12 yrs
	Reverse thinking	23%	34%	12 yrs
sandwiches, how many types	correct multiplication	90%	62%	
signpost, distance apart	correct addition	77%	78%	12 yrs
miles travelled with 7.4 gallons	correct multiplication	62%	53%	15 yrs
potatoes, cost for 0.52 kg	correct multiplication	38%	29%	15 yrs
students, how many per tutor	Correct division	94%	81%	

**Table A3.3: Results for questions on Place Value (Brown, 1981b)**

	Student Survey	CSMS Survey
Which is bigger? 30100 or 30095	97%	94%
Which is bigger? 7.06 or 7.5	97%	80%
Rounding of 0.28	83%	59%
Number nearest in size (place value)	57%	22%

**Table A3.4: Results for questions on Directed Numbers (Kuchemann, 1981)**

		Student Survey	CSMS Survey	CSMS Age
add +3 and +6	correct +9	78%	97%	14 yrs
	left blank or comments	16%		
multiplying -5 and -6	correct 30	44%	76%	14 yrs
	answer -30 or both 30 and -30	30%	12%	14 yrs
	left blank	16%		
add +2 and -4	correct -2	62%	82%	14 yrs
	left blank	22%		
- 8 plus + 12	correct 4	60%	-	
	left blank or finished	30%		
+ 3 x - 4	correct minus 12	44%	80%	14 yrs
	answer plus 12	6%	10%	
	left blank or finished	40%		



Table A3.5: *Results for questions on Percentage (Hart, 1981b)*

		Student Survey	CSMS Survey
42 out of 600 as a percentage	correct 7%	35%	39%
	correct method but not calculated	8%	
	left blank	16%	
6% of 250	correct 15	57%	45%
	a correct method	8%	
	left blank or finished	25%	
£20 reduced by 5%	correct 19	64%	39%
	just reduction found or method	10%	
	left blank or finished	9%	
10% of 7.75	correct 0.775	58%	
	correct but rounded or truncated	5%	
	correct method but unable to divide by 10	5%	
	multiplied by 10	4%	
	put zero in front or somewhere in middle	3%	
	doubled it 14.5	1%	
	removed the 7 so 0.75	5%	
	stayed same	1%	
	answer 5.75	1%	
	answer 6.75	3%	
	answer 7.6	1%	
	left blank	12%	
Write 25% as a fraction	correct one quarter	78%	
	correct but left as 25/100	6%	
	answer 0.25	6%	
	left blank	4%	
Find value before 4% increase	correct £20000 or correct method	17%	
	4% of 20800 subtracted £19968 inc place value errors	19%	
	other answers	23%	
	left blank or finished	41%	

Table A3.6: Results for questions on Ratio (Hart, 1981b)					
		No.	Student Survey	CSMS Survey	CSMS Age
Snakes, doubling	correct 4	76	99%	85%	all
	answer 3	1	1%		
	left blank	0	0%		
Snakes, trebling	correct 6	65	84%	72%	all
	answer 5	10	13%		
	answer 4	1	1%		
	answer 12	1	1%		
	left blank	1	1%		
Snakes x 1.5	correct 9	55	71%	50%	all
	answer 8	15	19%		
	answer 7	2	3%		
	answer 7 1/2	1	1%		
	answer 10	1	1%		
	answer 12	1	1%		
	answer 15	1	1%		
	left blank	1	1%		
Snakes x 2/3	correct 8	47	61%	50%	all
	answer 9	11	14%		
	answer 10	4	5%		
	answer 11	3	4%		
	answer 4	2	3%		
	answer 6	2	3%		
	answer 3	1	1%		
	answer 5	1	1%		
	answer 7	1	1%		
	answer 16	1	1%		
cards, 8 cost 48p, how much for 12 (x1.5)	correct 72 pence	53	69%		
	correct unitary method but minor numerical error	6	8%		
	1.5 times but numerical error	2	3%		
	48 multiplied by 12	2	3%		
	48 multiplied by 2	2	3%		
	other answers	4	5%		
	left blank	8	10%		
rock sample	correct 2 parts copper to 3 parts tin or multiples	28	36%	32%	all
	correct inverted 3 to 2 or multiples	4	5%		
	1 to 2 or reversed or multiples	7	9%		
	1 to 3 or reversed or multiples of	10	13%	20%	all
	1 to 5 or reversed or multiples	4	5%		
	1 to 6 or reversed or multiples	3	4%		
	1 to 15 or reversed or multiples	2	3%		
	2 to 5	1	1%		
	left blank	17	22%		
	can't read	1	1%		
rock sample harder	correct 16 to 9 or multiples	22	29%	13%	all
	correct inverted 9 to 16 or multiple	2	3%		
	3 to 8 or reversed or multiples	11	14%		
	1 to 5 or reversed or multiples	6	8%		
	8 to 15	2	3%		
	other answers, none repeated	11	14%		
	left blank	23	30%		

eggs , halving 12	correct 6	75	97%	95%	
	answer 7	1	1%		
	left blank	1	1%		
lemons, halving 2	correct 1	74	96%		
	answer 1/3 or 1 and half	2	3%		
	left blank	1	1%		
packet, halving a half	correct 1/4	73	95%		
	other answers	2	3%		
	left blank	2	3%		
eggs, 3/4 of 12	correct 9	49	64%	78%	
	answer 8	20	26%		
	other answers 4, 10, 18, 18, half	5	6%		
	left blank	3	4%		
lemons 3/4 of 2	correct 1 and half	63	82%	85%	
	answer 1 and quarter	3	4%		
	4/3, 5/2, 3/4, 3/4, 7/2, 1/2, 3, 3	8	10%		
	left blank	3	4%		
packet 3/4 of 1/2	correct 3/8	18	23%	26%	
	answer 1/3	22	29%		
	answer 3/4	11	14%		
	answer 2/3	4	5%		
	other answers	12	16%		
	left blank or can't read	10	13%		
20 spoonfulls for 8, how much for 6 (ie x3/4)	correct 15	43	56%	78%	all
	correct method or part	5	6%		
	answer 16	8	10%		
	answer 17	1	1%		
	answer 18	4	5%		
	other answers 11,12,13.5	4	5%		
	left blank	7	9%		
	finished	6	8%		
how much for 15	correct 37.5	29	38%		
	correct method	4	5%		
	answer 30	3	4%		
	answer 36	3	4%		
	answer 38	8	10%		
	answer 39	2	3%		
	answer 40	3	4%		
	10,15,18,3.75,32/3,300	6	8%		
	left blank	9	12%		
large or small	finished	9	12%		
	correct small and or working	36			
	large	12			
	left blank	11			
	finished	18			

## Appendix 3c - Questionnaire Excerpts

## Comparison of Cost of Items Questions

Table A3.7: Questionnaire Excerpts – Cost of Items Questions	
	<p>Question 18</p> <p>8 cards cost 48p. How much would 12 cost?</p>
F	<p>on 18 #241</p> <p>8 cards cost 48p. How much would 12 cost?</p> <p>72p</p> <p>48 24 72</p>
G	<p>24</p> <p>48</p> <p>72</p>
H	<p>Question 18</p> <p>8 cards cost 48p. How much would 12 cost?</p> <p>4 cards cost 24p ∴ 12 cards cost <math>3 \times 24p = [72p]</math></p> <p>Question 19</p> <p>72</p>
J	<p>Question 18</p> <p>8 cards cost 48p. How much would 12 cost?</p> <p>72p</p>
K	<p>Question 18</p> <p>8 cards cost 48p. How much would 12 cost?</p> <p>1 card = 6p</p> <p>12 cards = 72p</p> <p>12 x 6 = 72</p>
L	<p>Question 18</p> <p>8 cards cost 48p. How much would 12 cost?</p> <p>12 x 48 = 576</p> <p>576 ÷ 4 = 144</p> <p>144 x 12 = 1728</p>
	<p>Question 6</p> <p>6 cards cost 48p. How much would 11 cost?</p> <p>48 ÷ 6 = 8</p> <p>8 x 11 = 88p</p>

M	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $12 \times 8 = 96p$	<p>1 card 2p <math>\times 11 = 22p</math></p>
N	<p>Question 18 3 cards cost 48p. How much would 12 cost?</p> $3 \div 3 = 12$	<p><math>48 \div 3 = 16</math> <math>16 \times 12 = 192p</math></p>
P	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $8 \div 4 = 2$	<p><math>48 \div 4 = 12</math> <math>12 \times 12 = 144p</math></p>
Q	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $48 \div 8 = 6$	<p>6 cards cost 48p. How much would 11 cost?</p> $48 \div 6 = 8$ $8 \times 11 = 88p$
R	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $12 \div 8 = 1.5$ $1.5 \times 48 = 72p$	<p>Question 19 6 cards cost 48p. How much would 11 cost?</p> $48 \div 6 = 8$ $8 \times 11 = 88p$
T	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $48 \div 8 = 6$ $6 \times 12 = 72p$	<p>Question 19 6 cards cost 48p. How much would 11 cost?</p> $48 \div 6 = 8$ $8 \times 11 = 88p$

U	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $(48 \div 8) \times 12$	<p>Question 19 6 cards cost 48p. How much would 11 cost?</p> $(48 \div 6) \times 11$
V	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $\frac{48}{8} = 6 \quad \therefore 12 \times 6 = 72p$	<p>Question 19 6 cards cost 48p. How much would 11 cost?</p> $\frac{48}{6} = 8 \quad \therefore 11 \times 8 = 88p$
W	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $48 \div 8 = 6p \quad 12 \times 6 = 72p$	<p>Question 19 6 cards cost 48p. How much would 11 cost?</p> $48 \div 6 = 8p \quad 11 \times 8 = 88p$
X	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $72$	<p>Question 19 6 cards cost 48p. How much would 11 cost?</p> $6 \div 48 = 8$ $11 \times 8 = 88$
Y	<p>Question 18 8 cards cost 48p. How much would 12 cost?</p> $\begin{array}{r} 8 \overline{)48} \\ \underline{16} \phantom{0} \\ 32 \phantom{0} \\ \underline{24} \\ 8 \end{array}$	<p>Question 19 6 cards cost 48p. How much would 11 cost?</p> $\begin{array}{r} 6 \overline{)48} \\ \underline{12} \phantom{0} \\ 36 \phantom{0} \\ \underline{24} \\ 12 \end{array}$ <p>8p per card <math>\times 11</math> 88 pence.</p>



## **Appendix 4 - Monitoring of Anxiety and Attitude**

### **(Excerpt from Pilot Study report, pp. 29-30)**

One purpose of the Initial Study was to monitor anxiety caused by completion of the questionnaire, so it was decided to include an attitude scale in the October questionnaire similar to that used by Evans (2000). Evans asked people to rate anxiety levels for different activities on a 1-7 scale. My survey used just 24 activities (See Appendix 1c), including 'completing this questionnaire' and activities that had taken place during student induction . The remaining activities were selected and adjusted from Evans' list. The activity, 'Being asked a question by a tutor in an English class' was used twice. Table A4.1 shows the ratings for the repeated activity.

Table A4.1: <i>Comparison of anxiety levels for the same activity.</i> <i>'Being asked a question by a tutor in an English class'</i>		
RESPONSE	NO. OF STUDENTS	%
Recognised activities the same	8	9
same rating	52	56
ratings differ by 1	21	23
ratings differ by 2 + (mark not tick)	9 + (1)	10 + (1)
ratings differ by 3	2	2
<b>TOTAL</b>	<b>93</b>	<b>100 %</b>

I suggest that a rating difference of 1 might be considered acceptable, but a difference of 2 or 3 by 13% of people indicates a high level of unreliability in this particular survey. The selection of activities for the list also appeared flawed. It had been hoped that certain activities, like selecting a film, would form a low anxiety baseline for comparison, but no activity was identified in this category by a majority of students. With these concerns, there seemed little value in analysing the remaining activity ratings in detail at this stage. 'Completing the questionnaire', was the last activity on the list, see Table A4.2. If this ranking

were reliable, it would imply that the survey process did not raise unacceptable levels of anxiety for the majority of students. However, it must be noted that this activity was rated prior to the completion of the maths questions.

Table A4.2: <i>Anxiety ratings for 'Completing this questionnaire'</i>	
RESPONSE	NO. OF STUDENTS
Neither relaxed nor anxious	16
Very relaxed, relaxed or fairly relaxed	71
A little anxious	3
Moderately anxious	2
Very Anxious	1
<b>TOTAL</b>	<b>93</b>

### ***Implications for future work***

The anxiety survey was intended, primarily, to monitor levels of anxiety caused by completing the preliminary questionnaire. It was not expected to extend use of such surveys into later research, unless they proved exceptionally valuable.

Future reliability could be improved by better design, more questions, a more balanced set of activities and a greater depth of reading or understanding by the researcher. However, flaws in the design and use of this tool made the results less reliable than the skills of an experienced teacher in identifying severe anxiety. Study in other areas seems of more direct value to this research.

Although I argue that the anxiety section had little value as a monitoring tool, it did provide a non-threatening survey introduction and, for some students, may have reduced anxiety by formalising the notion that other people were also anxious about maths. Removal of this section in future questionnaires would allow more time to be allocated to other areas, but some alternative preliminary questions which served these two additional purposes might need to be included. A debrief anxiety question at the end might also be of value.



## **Appendix 5 - Mature students – Attitudes and Beliefs**

### ***Overview***

Discussion in Chapter 2 and particularly in Section 2.1 has drawn a picture of Foundation Programme learners. A number of claims were made about mature learners in general, which I suggested were applicable to the Foundation student cohort in certain ways. Some of these ideas had lead to the development of the 'Foundation Centre ethos' which was embedded in Foundation Centre practice (Section 2.1). Perhaps more importantly for this research, this notion of mature learners having a particular perspective or set of needs (Knowles, 1980; Benn, 1997) was fundamental in the development of research questions and underpinned the design of this study.

The primary function of the interviews was to provide possible triangulation for ideas emerging from the questionnaire and to provide data for consideration of the research questions. However, through the search for resonance, groups of themes emerged, many of which appeared to relate to these notions of an adult learner. Additionally, data on anxiety had been collected through the questionnaires. Together, these sources of data provided valuable additional support for the validity of the claims about foundation students on which this study was based and are therefore considered in the following sections.

### ***Adults as self-directed learners***

Developing from notions of androgogy (Knowles, 1980) and writings particularly from Benn (1997) and Merriam *et al.* (2007), Section 2.1 put forward the notion of adults as self-directed learners. It was suggested that foundation adults were likely to expect teaching to be subject centred in line with their perceived purpose

in returning to study and, further, that they might have clear expectations about what a teacher-led experience should be possibly based on previous experiences of being taught (Duffin and Simpson, 2000).

Discussions in interviews showed that all five students had a clearly identified purpose for studying on the Foundation Programme. All five mentioned their route at some point in relation to what they were learning. It was apparent that this purpose guided their belief about what they should be taught (or not) and, therefore, their interaction with it.

Brenda thought some areas would be useful:

*'so I think in terms of going on for medicine I think it will be helpful particularly things like percentages ... and relative quantities in things like in chemistry.'*

and later linked this idea of usefulness for the future with the reason for learning:

*'that's a skill I've developed and will use in the future whereas maths at school, you wonder why you're doing it.'*

Ann indicated the use of judgement :

*'With the normal distribution and the standard deviation and all that I would have liked a lot more time on that because I felt like this is what we really need to know ... this stuff (older maths topics) was good and it builds your confidence and what have you but I really need to know that for my degree.'*

She had clearly decided that certain statistics topics would be useful for her degree. The partial rejection of the value of other topics might have reflected her belief that she was there to learn new things. This was supported by her later debate about trigonometry which was new to her. She did not immediately reject it

even though it was less obviously useful than statistics or percentage:

*'I don't know how much impact that has on the course later on. I don't know if we are going to have to work out that kind of stuff, if I'm going to need this.'*

Diane too was judging against purpose. Some things might be useful:

*'going over that knowing what it was to do the psychology project .... so I kind of could see that it was where we were heading towards.'*

but some things might not!:

*'when you had the triangle and you cut them off, and put them on a line to prove that it was 180 degrees and things like that I just thought do we really need to do these in psychology?'*

Whilst this is a rejection of things not seen as relevant to her course, the specific identification of this activity, the cutting corners off triangles, supported my suggestion in Section 2.1, that some adults resist the use of innovative practices and games. Comments by Diane also supported my suggestion that students have clear ideas about how they should be taught. The excerpt below was a clear rejection of 'starting again' on this occasion:

*'some of the things I did think, I really don't want to do 20 of these questions, because I can do the first one and last one so obviously I'm good at doing it.'*

All five students showed evidence of self-motivation that was clearly driven by their wish to proceed further. As Ann put it the:

*'reason that I'm here is not to get like 41% which would be great but ... I want to understand this because I know that I'm going to have to use it later on.'*

or Christine:

*'you're working hard and you know what it's for, and I'm thinking I really want to do a good job.'*

This self-motivation was accompanied by an ownership of learning, a recognition that they were responsible and needed to be proactive. This ranged from the simple statement by Elaine:

*'I still can't get my head around percentages, I shall have to sit and work on them.'*

To a much deeper reflection by Ann about needing to adjust her learning methods:

*'Sometimes you can do the new stuff on the day and you can think yeah yeah yeah, and you go away and if you don't do anything with it for a while, you may well forget it but some things are easier to gain than others and for me that takes a lot more work to get it into the noggin.'*

However, when she states:

*'I would have liked all of them to be much earlier so I could at least put in the extra work when I didn't grasp it all straight away.'*

she is taking responsibility for the need to put in her own effort but at the same time indicating her desire for control of what should be taught and when.

### ***Adult Mathematics Anxiety, Panic and Shame***

Discussions in Section 2.1 made two particular claims: the first that some adults might have high levels of anxiety (Bibby, 2002; Buxton, 1981; Coben, 2000; Evans, 2000; Karsenty, 2004; Klinger, 2007), the second that levels of anxiety might have a major impact on performance.

In the interviews, all except Christine referred in some way to anxiety or to lack of confidence at the beginning of the course. Ann said that she had *'never been*

*confident with maths*'. Brenda talked about a reduction in 'fear' as the Foundation maths course progressed and remembered:

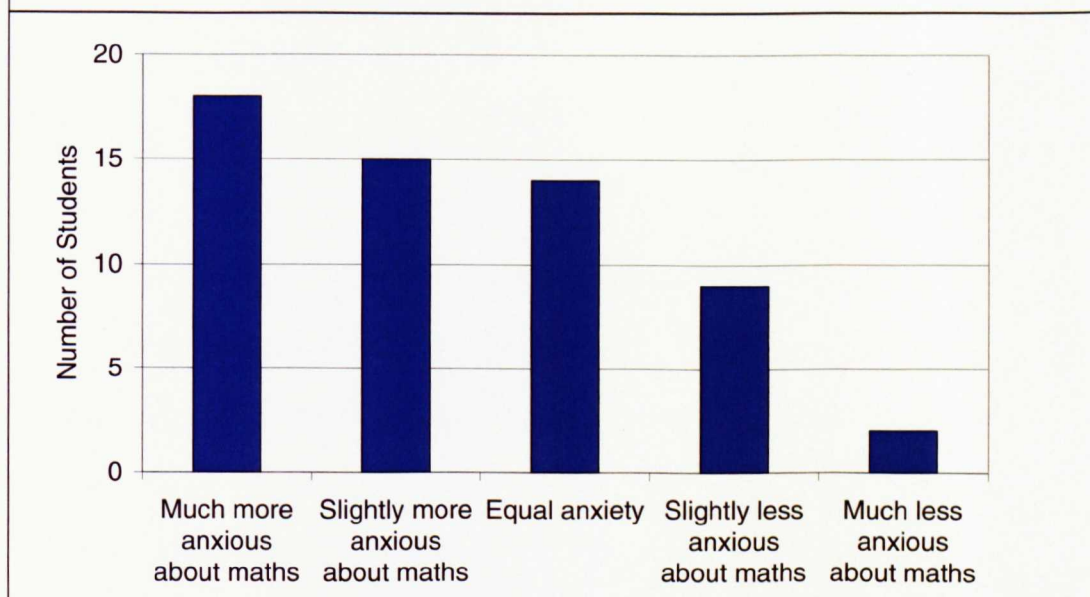
*'I was very, quite anxious about it at the beginning of the year and was thinking, maths, I won't be able to do it.'*

Elaine had a more extreme reaction and refers to panic and being 'scared to death' and later to the notion of guilt or shame (Bibby, 2002):

*'I always felt awful when my kids were growing up because I couldn't help them.'*

In the questionnaires, students were asked to rate their anxiety levels about future study and future mathematics study on a scale of 1-10. Figure A5.1 shows the results. The bar chart shows that, in general, more students were anxious about

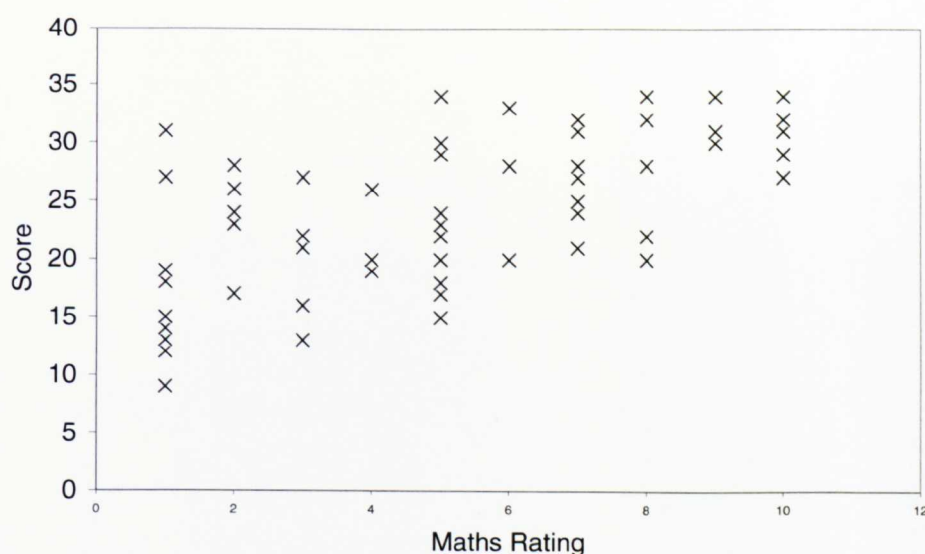
Figure A5.1: Bar Chart comparing Anxiety about Maths study with Foundation study in general. (Home students Oct 2007)



mathematics than their overall study, some giving as much as 6 rankings difference. These results support the claim that some students might have high levels of anxiety, although clearly some do not.

To consider the claim that anxiety levels might have an impact on performance, anxiety and score were considered together. The results are shown in Figure A5.2. The scattergram shows that there was no clear correlation between anxiety and score, but some results were of particular interest. It could be seen that all students who identified themselves as 'not at all anxious' gained high marks.

Figure A5.2: A Scattergram to compare scores with maths anxiety rating  
(Home students 2007)



Those gaining the lowest marks identified themselves as 'very anxious' but it cannot be distinguished from these results alone whether it was the anxiety that caused the low performance as in a Buxton (1981) panic type reaction or whether it was lack of mathematical skill that caused the low performance and it was an awareness of this lack that caused the anxiety. It should not be assumed that those identified as anxious always do badly, some of those labelled as 'very anxious' actually performed very well supporting Evans (2000) suggestion that anxiety might sometimes improve performance.

### **Adult self-image**

A number of authors (Bibby, 2002; Coben, 2000; Klinger, 2007) have identified

instances of negative self-image amongst mature students. Klinger (2007) particularly identified this in his study of access and foundation students. One of the more obvious signs for this is the use of self-denigration (Bibby, 2002) and some examples of this could be found within the interview transcripts.

All except Ann used occasional self-denigrating comments. For Christine, these referred to her actions:

*'Sorry about my answers.'*

For Diane these were more personal:

*'but I am generally always wrong.'*

However, these were also balanced by some more positive comments. Christine clearly had a fairly positive school experience and said:

*'I remember doing stuff like this at school where I was good at it.'*

Diane seemed to reassure herself with:

*'I like maths anyway and I'm not really that terrible at it'*

Elaine with the confidence rating of 1 (Very anxious) and no recorded qualification in maths had by far the most negative self-image in the group and used some particularly strong statements of self-denigration. She refers to herself as: *'thick and stupid'* on a three occasions. Her low self-esteem is clear when she qualifies her answers with:

*'but there again it might not, I'm not very good with fractions'* and

*'I'm probably totally wrong because like I said I'm rubbish at maths'*

and in her comments about her concerns when starting the Foundation course:

*'I'm never going to get through this, how am I going to do it'*

This last statement in particular shows a very low level of self-efficacy (Bandura, 1997) or perception of her own capabilities. In line with Klinger's (2007) findings,

her negative self-image and low self-efficacy clearly dates from a poor early learning experience and the influence of a 'significant other' (Coben, 2000, Karsenty, 2004). Indeed she suggests that the thick and stupid label was originally given to her by a primary teacher. She describes maths lessons as occasions when she and another girl were given a book and told to get on with it:

*'Normally I was sat in the corner with a reading book that was my maths lesson.'*

I have previously suggested that improving the negative self-image and levels of self-efficacy of adults has the potential to be one of the easiest and most effective ways to improve learning for some adults. Structuring teaching to incorporate experiences of success, encouraging mentoring and role models (Bandura, 1997) and increased determination returning as an adult (Benn, 1997) all had the possibility of raising self-efficacy, and this in turn would increase the motivation, performance and ultimate success of students (Bandura, 1997). Elaine's later remarks appeared to support this idea:

*'I am absolutely chuffed to ribbons because I couldn't do algebra,'*

and :

*'I really find it strange when I say I enjoyed maths I really did, I enjoyed the learning.'*

She now recognised that she was not necessarily to blame. She noted:

*'I now don't think I'm thick and stupid I just had a bad teacher.'*

and later:

*'all we really needed was a bit of encouragement and a bit of help and the light might have gone on.'*



## **Adult views of mathematics**

It was previously suggested that the abstract nature of mathematics (Coben, 2000) and its perceived power as a gatekeeper (Bibby, 2002) were linked with some of the negative emotions involving it. Elaine refers to maths being a *'like a brick wall'* and later in the interview as *'a foreign language'*.

Ann's memories of probability also seemed to support this view:

*'that was just another abstract thing floating about that I thought ok I'll try and remember that hopefully I'll come to understand it.'*

or with algebra:

*'and he said to me 'well this letter can be any letter and it can be any number' and I thought 'well that's just silly, that's just ridiculous, you could just pluck any number out of the air, 'a+b=x' well of course it does but it's meaningless.s'*

and later:

*'it didn't gel properly at school, I thought they were just having a laugh with us, thought they were just messing with my mind.'*

For Elaine, finally making sense of the abstract transferred her view of mathematics from 'gatekeeper' to 'gateway' as she visualised a new future:

*'Now I understand where it is all coming from ... just being something written on a piece of paper you could actually see the maths itself transferred into a document and I thought oh wow yeah I could be a statistician.'*

## **A different adult perspective?**

Within their interviews all five students identified their learning experience as an adult as different from their learning experience in the past. This was not

unexpected, given the different learning environment, different subject material and, perhaps, the Foundation ethos. Identifying whether students actually felt themselves to be different was difficult. However, I suggest that the extracts below from two different students are strikingly similar. Both appear to attribute a change in their learning success, at least in part, to a change within themselves.

From Christine:

*'I don't know if it's because older and more open to these ideas or if it was just something I was taught at school and I thought nah, don't want to know ... it just didn't fit with how I worked, and your workings just seemed to fit better with the way I worked.'*

From Brenda:

*'I don't know if that's age or confidence on my part but there were a lot of things that were gone through the same way as when we were at school but they were more accessible.'*

Note that Christine appeared to identify a change in working or method which also helped. Brenda appeared to suggest that methods taught in school were the same as methods taught in Foundation, i.e. 'one right method'.